# Quantum vacuum energy density and unifying perspectives between gravity and quantum behaviour of matter

Davide Fiscaletti[a], Amrit Sorli[b]

[a]SpaceLife Institute, S. Lorenzo in Campo (PU), Italy
corresponding author, email: spacelife.institute@gmail.com
[b]SpaceLife Institute, S. Lorenzo in Campo (PU), Italy
Foundations of Physics Institute, Idrija, Slovenia
email: sorli.amrit@gmail.com

ABSTRACT. A model of a three-dimensional quantum vacuum based on Planck energy density as a universal property of a granular space is suggested. This model introduces the possibility to interpret gravity and the quantum behaviour of matter as two different aspects of the same origin. The change of the quantum vacuum energy density can be considered as the fundamental medium which determines a bridge between gravity and the quantum behaviour, leading to new interesting perspectives about the problem of unifying gravity with quantum theory.

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## 1 Introduction

The standard interpretation of phenomena in gravitational fields is in terms of a fundamentally curved space-time. However, this approach leads to well known problems if one aims to find a unifying picture which takes into account some basic aspects of the quantum theory. For this reason, several authors advocated bfdrde955Ddyzerms)-4407()28(t)-47attu1(tati)1(on)-1(a)1(l)-426(ln)27(to)-1(m elasticity" of space [1] (the reader can also see the reference [2] for a review of this concept), Haisch's and Rueda's model [3] regarding the interpretation of inertial mass and gravitational mass as effects of the electromagnetic quantum vacuum, Puthoff's polarizable vacuum model of gravitation [4] and, more recently, a model developed by Consoli based on ultra-weak excitations in a condensed manifold in order to describe gravitation and Higgs mechanism [5-7].

Under the construction of all of these models there is probably one underlying fundamental observation: as light in Euclid space deviates from a straight line in a medium with variable density, an "effective" curvature might originate from the same physical flat-space vacuum. In particular, in Consoli's model, the physical vacuum is represented by a superfluid medium – a Bose condensate of elementary spinless quanta – whose long-range fluctuations, on a coarse-grained scale, resemble the Newtonian potential, allowing the first approximation to the metric structure of classical general relativity to be obtained. More precisely, taking into consideration the long-wavelength modes, gravity is induced by an underlying scalar field $s(x)$ describing the density fluctuations of the vacuum (that in weak gravitational fields, on a coarse-grained scale, is identified with the Newtonian potential). The effect of this scalar field (and thus of the density fluctuations of the vacuum) is to determine a re-scaling of the masses. Moreover, Consoli's model advocates a phenomenon of symmetry breaking consisting in the spontaneous creation from the empty vacuum of elementary spinless scalar quanta and provides a simple unified picture of the scalar field in terms of the density of the elementary quanta (treated as hard spheres) and of their scattering length.

On the other hand, some relevant current research introduce the interesting perspective to interpret gravity and quantum behaviour as two different aspects of a same source, of a same coin. In particular, A. Shojai and F. Shojai recently developed an interesting toy model which, studying the behaviour of particles at spin 0 in a curved space-time, demonstrates that quantum potential provides a contribution to the curvature that is added to the classic one and reveals deep and unexpected connections between gravity and the quantum phenomena [8, 9]. By the investigation of the coupling of purely gravitational effects and purely quantum effects of a particle in a general background space-time metric, this approach can obtain a fundamental equivalence of quantum effects of matter and a curved space-time. By the analysis of the quantum effects of matter in the framework of bohmian mechanics, A. Shojai's and

F. Shojai's model shows that the motion of a particle (of spin zero) with quantum effects is equivalent to its motion in a curved space-time. A fundamental geometrization of quantum aspects of matter emerges: the quantum effects of matter as well as the gravitational effects of matter have geometrical nature and are highly related in the sense that the quantum potential appears as the conformal degree of freedom of the space–time metric and its presence is equivalent to the curved space-time. In other words, on the basis of F. Shojai's and A. Shojai's model, one can say that there is a dual aspect to the role of geometry in physics. The space-time geometry sometimes looks like what we call gravity and sometimes looks like what we understand as quantum behaviours: the particles determine the curvature of space-time and at the same time the space-time metric is linked with the quantum potential which influences the behaviour of the particles.

In this article the aim of the authors is to propose a model of a three-dimensional granular background, whose most universal physical property is the Planck energy density, which allows us to obtain, re-read and somewhat unify the above mentioned important foundations and results of Consoli's model and of F. Shojai's and A. Shojai's model. The article is structured in the following manner. In chapter 2 we will review some relevant theoretical results regarding the description of space through quantum gravity and quantum vacuum. In chapter 3 we will enunciate the starting-points and the postulates of our model. In chapter 4 we will analyse the mathematical formalism of our model and its fundamental results as regards the treatment of gravitation. In chapter 5 we will illustrate the interpretation of the quantum behaviour of matter and we will conclude with some considerations about the unifying picture of gravity and quantum behaviour inside our model.

## 2 An overview of quantum gravity and results about the quantum vacuum

Loop quantum gravity suggests that in the quantum-gravity domain physical space is composed by elementary grains, a net of intersecting loops, having approximately the Planck size [10-12]. On the basis of the results of loop quantum gravity, quanta of space having the size of Planck's volume $V = (l_p)^3$, where $l_p = \left(\hbar G/c^3\right)^{1/2}$ is Planck's length, $\hbar$ is Planck's reduced-constant, $G$ is the universal gravitation constant and $c$ is the speed of light, can be considered as the fundamental constituents of the universe. In the kinematics of loop quantum gravity the geometric operators representing area, angle, length and volume have thus discrete spectra [13-20]. The geometry associated with spin network states of loop quantum gravity is characterized by a discrete quantized three-dimensional (3D) metric. This discreteness of quantum geometry at the Planck scale can be considered as a genuine property of space, independent of the strength of the actual gravitational field at any given location.

The Planck volume as it emerges from loop quantum gravity implies that, at a fundamental level, quantum space has to be described by three spatial dimensions, namely it is physically three-dimensional. By following Rovelli in his book *Quantum Gravity* [21], let us consider a classical macroscopic 3D gravitational field which determines a 3D metric $g_{\mu\nu}$ and in this metric let us fix a region R of area S with a size much larger than $l >> l_P$ and slowly varying at this scale. A spin network state $|S\rangle$, if is an eigenstate of the volume operator $V(R)$ (and of the area operator $A(S)$), with eigenvalues equal to the volume of R (and of the area of S) determined by the metric $g_{\mu\nu}$, up to small corrections in $l/l_P$, is called a weave state of the metric $g$.

Several weave states were constructed and analysed in the early days of loop quantum gravity, for various 3D metrics, including those for flat space, Schwarzschild and gravitational waves. Weave states have played an important role as regards the historical development of loop quantum gravity. In particular, they provided an explanation of the emergence of the Planck-scale discreteness. The intuition was that a macroscopic geometry could be built by taking the limit of an infinitely dense lattice of loops, as the lattice size goes to 0. With increasing density of loops, the eigenvalue of the operator turned out to increase.

In order to define a weave on a 3D manifold with coordinates $x$ that approximates the flat 3D metric $g^{(0)}_{\mu\nu}(x) = \eta_{\mu\nu}$, one can build a spatially uniform weave state $|S_{\mu_0}\rangle$ constituted by a set of loops of coordinate density $\rho = \mu_0^{-2}$. The loops are then at an average distance $\mu_0$ from each other. By decreasing the "lattice spacing" $\mu_0$, namely by increasing the coordinate density of the loops, one obtains

$$A(S)|S_\mu\rangle \approx \frac{\mu_0^2}{\mu^2}\left(A\left[g^{(0)}, S\right] + O\left(\frac{l}{l_P}\right)\right)|S\rangle \tag{1}$$

which indicates an increasing of the area. Since

$$\frac{\mu_0^2}{\mu^2}A\left[g^{(0)}, S\right] = \left(A\left[\frac{\mu_0}{\mu}g^{(0)}, S\right] + O\left(\frac{l}{l_P}\right)\right) = A\left[g^{(\mu)}, S\right] \tag{2}$$

the weave with increased loop density approximates the metric

$$g^{(\mu)}_{\mu\nu}(x) = \frac{\mu_0^2}{\mu^2}\eta_{\mu\nu} \tag{3}$$

At the same time, however, the physical density $\rho_\mu$, defined as the ratio between the total length of the loops and the total volume, determined by the metric $g^{(\mu)}$, remains $\mu_0$, irrespective of the density of the loops $\mu$ chosen:

$$\rho_\mu = \frac{L_\mu}{V_\mu} = \frac{(\mu_0/\mu)\,L}{(\mu_0/\mu)^3\,V} = \frac{\mu^2}{\mu_0^2}\rho = \frac{\mu^2}{\mu_0^2}\mu^{-2} = \mu_0^{-2} \tag{4}$$

Equation (4) implies that, if $\mu_0$ is not determined by the density of the loops, it must be given by the only dimensional constant of the theory, namely the Planck length:

$$\mu_0 \approx l_P \tag{5}$$

By substituting equation (5) into equation (3), the metric approximated by the increased loop density becomes

$$g^{(\mu)}_{\mu\nu}(x) = \frac{l_P^2}{\mu^2}\eta_{\mu\nu} \tag{6}$$

The physical meaning of the approach based on equations (1)-(6) is that, in loop quantum gravity, a smooth geometry cannot be approximated at a physical scale lower than the Planck length. Each loop carries a quantum geometry of the Planck scale: more loops give more size, not a better approximation to a given geometry.

Another significant feature of the texture of the weaves of the fundamental quantum geometry of the Planck scale is its holographic nature. In this regard, in the paper [22] Gambini and Pullin showed that from the framework of loop quantum gravity in spherical symmetry an holography emerges in the form of an uncertainty in the determination of volumes that grows radially. Even more interesting is the holographic model of fundamental spacetime foam recently proposed by Jack Ng in the papers [23-26], in which quantum fluctuations of spacetime manifest themselves in the form of uncertainties in the geometry of spacetime and thus the structure of spacetime foam can be inferred from the accuracy with which we can measure its geometry. By considering a spherical volume of radius $l$, the average minimum uncertainty in its measurement is

$$\delta l \geq \left(2\pi^2/3\right)^{1/3} l^{1/3} l_P^{2/3} \tag{7}$$

Ng's approach is a holographic model in the sense that, dropping the multiplicative factor of order 1, since on the average each cell occupies a spatial volume of $ll_P^2$ , a spatial region of size $l$can contain a maximum number of $l^3/\left(ll_P^2\right)=(l/l_P)^2$ cells and thus of bits of information, that is allowed by the holographic principle [27-32] which implies that, although the world around us appears to have three spatial dimensions, its contents can actually be encoded on a two-dimensional surface, like a hologram. In Ng's model, as a consequence of the holographic principle, the physical degrees of freedom of the spacetime foam, at the Planck scale, must be considered as infinitely correlated, with the result that the spacetime location of an event may lose its invariant significance. In other words, in virtue of its holographic nature, the spacetime foam gives rise to non-locality. This argument is also supported by the observation that the long-wavelength (hence "non-local") "particles" constituting dark energy in the holographic spacetime foam cosmology obey an exotic statistics which has attributes of non-locality [33]. In Ng's model, since the holographic principle is believed to be an important ingredient in the formulation of quantum gravity, it is just the non-local features of the spacetime foam that make it easier to incorporate gravitational interactions in the theory. In synthesis, in the light of relevant current research, the suggestive perspective is opened that the fundamental space-time background of processes at the Planck scale is a non-local, holographic quantum foam characterized by a deformation of the geometry.

On the other hand, the laws of quantum mechanics applied to electromagnetic radiation imply the existence of a fundamental level, a background 'sea' of electromagnetic zero-point energy that can be called the electromagnetic quantum vacuum. In this picture, space can be described in terms of fluctuations which are just the consequence of the energy density of the electromagnetic quantum vacuum (and of Heisenberg's uncertainty principle). The electromagnetic quantum vacuum can be considered as a real dynamic entity, able to manifest energetic fluctuations, not directly observable but significant in the investigation of the behaviour of physical systems in the sense that it can produce real and observable e□ects on them. Moreover, the development of modern quantum field theories (in particular, the quantum electrodynamics, the Weinberg-Salam-Glashow theory of electroweak interactions, and the quantum chromodynamics of strong interactions) brought important contributions as regards the nature of the physical vacuum on the different energy scales. In the last decade, the notion of a physical vacuum have come into wide use also in cosmology [34-37], in connection with the standard model of the dynamics of the universe, at whose root lie the Friedmann equations of the general theory of relativity, with “dark energy” that accounts for 73% of the entire energy of the universe.

On the light of quantum field theories and cosmology, the physical vacuum can be regarded as a real relativistically invariant quantum arena (a kind of quantum fluid) filling out the whole universal space and corresponding to the lowest energy state. The real particles such as electrons, positrons, photons, hadrons etc. as well as all macroscopic bodies are quantum wave-like excitations of this medium endowed with certain quantum numbers ensuring their relative stability. Moreover, the discovery of dark energy in cosmology and the estimate of its relatively low value (see the review paper [38] and references therein) raises the issue about the mechanism of radical reduction of the vacuum energy density with respect to the predictions of local quantum field theories.

Since 1994, Haisch and Rueda suggested that the inertial mass can be interpreted as an effect of the electromagnetic quantum vacuum: according to their results, the amount of electromagnetic zero-point energy instantaneously transiting through an object of a given volume and interacting with the quarks and electrons in that object is what constitutes the inertial mass of that object [39]. In this model inertia emerges thus as a kind of acceleration-dependent electromagnetic quantum vacuum drag force acting upon electromagnetically interacting elementary particles (electrons and quarks). Successively, Rueda and Haisch showed that the result for inertial mass can be extended to passive gravitational mass. In their article of 2005 *Gravity and the quantum vacuum inertia hypothesis*, they showed that, in the quantum vacuum inertia hypothesis, inertial and gravitational mass are not merely equal, they prove to be the identical thing: inertial mass arises upon acceleration through the electromagnetic quantum vacuum, whereas gravitational mass — as manifest in weight — results from what may in a limited sense be viewed as acceleration of the electromagnetic quantum vacuum past a fixed object [3].

Already since the second half of the 60s several authors have proposed the idea that a vacuum energy is ultimately responsible for gravitation. In [1] Sakharov, on the basis of the work of Zeldovich [40], proposed a model in which a connection is drawn between Hilbert-Einstein action and a quantum vacuum intended as fundamental arena of physical reality. This led to a view of gravity as "a metric elasticity of space". Following Sakharov's idea and using the techniques of stochastic electrodynamics, Puthoff proposed that gravity could be construed as a (long range) form of the van der Waals force [41]. Although interesting and stimulating in some respects, Puthoff's attempt to derive a Newtonian inverse square force of gravity proved to be unsuccessful [42-45]. An alternative approach has been recently developed by Puthoff [4] that is based on earlier work of Dicke [46] and of Wilson [47]: a polarizable vacuum model of gravitation. In this representation, gravitation comes from an effect by massive bodies on both the permittivity, $\varepsilon_0$, and the permeability, $\mu_0$, of the vacuum and thus on the velocity of light in the presence of matter. That is clearly a theory alternative to general relativity, since it does not involve actual curvature of spacetime. On the other hand, since spacetime curvature is by definition inferred from light propagation in relativity theory (the bending of starlight), the polarizable vacuum gravitation model may be considered as a pseudo-metric theory of gravitation, since the effect which the variation in the dielectric properties of the vacuum near massive objects imposes on light propagation is approximately equivalent to general-relativistic spacetime curvature, as long as the fields are sufficiently weak. In the weak field limit, the polarizable vacuum model of gravitation replicates the results of general relativity, including the classic tests (gravitational redshift, bending of light near the Sun, advance of the perihelion of Mercury). Differences appear in the strong-field regime, which should lead to interesting tests between the two theories.

Taking into account the results of loop quantum gravity regarding the granular structure of space, Haisch's and Rueda's model regarding the interpretation of inertial mass and gravitational mass as an effect of the electromagnetic quantum vacuum, and Puthoff's polarizable vacuum model of gravitation, in the next chapters we will introduce a model of the gravitational interaction in which a three-dimensional universal granular space is the fundamental medium. As we will show, a central idea that underlies our model is that, at a fundamental level, gravity-space can be described by elementary entities having the size of Planck's volume and that gravity results from a sort of polarization, or better a shadowing of the gravitational space produced by the presence of a massive object (and associated with changes of the energy density of quantum vacuum).

## 3 The postulates of the three-dimensional quantum vacuum energy density model

The Planckian metric is the starting-point in order to define a quantum vacuum energy density intended as fundamental property of space. This can be shown by starting with a semi-classical argument derived from orbital mechanics, and then by imposing on it the necessary constraints of relativity and of quantum mechanics. A body within a circular orbit experiences a centripetal acceleration of $v^2/r$, derived from a gravitational force per unit mass, $Gm/r^2$ where $G$ is universal gravitation constant. If the velocity $v$ becomes equal to the speed of light $c$, then the maximum acceleration possible is $c^2/r = Gm/r^2$. From this equality one can easily obtain Schwarzschild's radius for a given mass $m$:

$$r_S = \frac{Gm}{c^2} \tag{8}$$

Now, Heisenberg's uncertainty principle requires that the position $x$ of an object and its instantaneous momentum $p$ cannot be known at the same time to a precision better than the amount imposed by the quantum of action, $\Delta x \Delta p \geq \hbar$. By expressing $\Delta p$ as $mc$, we obtain in the limiting case Compton's radius:

$$r_C = \frac{\hbar}{mc} \tag{9}$$

which is understood as the minimum possible size of a quantum object of mass $m$.

The laws of physics should be the same, regardless of the size of an object, so it makes sense to assume that the limits $r_C = \frac{\hbar}{mc}$ and $r_S = \frac{Gm}{c^2}$ must apply for all limiting cases, whether we are dealing with black holes or elementary particles. By equating this minimum quantum size for an object of mass $m$ with Schwarzschild's radius for that same object, $\frac{Gm}{c^2} = \frac{\hbar}{mc}$, we obtain Planck's mass limit:

$$m_P = \sqrt{c\hbar/G} \tag{10}$$

Compton's radius of the Planck's mass is then the shortest possible length in space, Planck's length:

$$l_P = \sqrt{\frac{\hbar G}{c^3}} \tag{11}$$

This means that, because of the uncertainty relation, a Planck's mass cannot be compressed into a volume smaller than the cube of a Planck's length. A Planck's mass contained within a Planck's volume $l_p^3$ therefore represents the maximum density of matter that can possibly exist:

$$\rho_P(m) = \frac{c^5}{\hbar G^2} \tag{12}$$

Following Einstein's most popular equation:

$$W = mc^2 \tag{13}$$

this mass density is equivalent to an energy density of:

$$\rho_P(W) = \frac{c^7}{\hbar G^2} \tag{14}$$

Likewise, the maximum energy that can be contained within a volume $l_p^3$ is the Planck energy:

$$W_P = m_p c^2 \approx 1,98 \cdot 10^9 J \tag{15}$$

Consequently the speed of light limit, $c$, together with the minimum quantized space $l_p^3$ constrain the highest oscillations that can be sustained in space:

$$\omega_P = \frac{W_P}{\hbar} = \sqrt{\frac{c^5}{\hbar G}} \approx 1,9 \cdot 10^{43} rad/s \tag{16}$$

All these relations represent the fundamental limits of nature, and thus the natural units of measure. By using these units it is possible to calculate the total average volumetric energy density, owed to all the frequency modes possible within the visible size of the universe. Because we are averaging over all possible oscillating modes, it is proper to express it as the square root of the squared energy density given by (14):

$$\rho_P = \sqrt{\frac{c^{14}}{\hbar^2 G^4}} \approx 4,641266 \cdot 10^{113} J/m^3 \tag{17}$$

Therefore, on the basis of Planck's metric, we can conclude that the maximum energy density characterizing the minimum quantized space constituted by Planck's volume is given by relation (14) and this defines an universal property of space.

Here, starting from the idea that space has a granular structure at the Planck scale and from the Planckian metric above mentioned, we suggest a model of a three-dimensional (3D) quantum vacuum – composed by elementary packets of energy having the size of Planck's volume – defined by an energy density which, in the absence of matter, is maximum and is given by the total volumetric energy density (17) (which can be expressed also as

$$\rho_{pE} = \frac{m_p \cdot c^2}{l_p^3} = 4,641266 \cdot 10^{113} \frac{Kg}{ms^2} \tag{18}$$

and thus can be defined as Planck energy density).

While in Consoli's model the physical vacuum is represented by a superfluid medium, a Bose condensate of elementary spinless quanta, in our model the background of processes, the physical vacuum is a 3D granular structure characterized by a energy density corresponding to the Planck energy density (intended as a universal property of nature). While Consoli's model defines the physical vacuum by starting from the Bose condensates of particle physics (and, as a consequence of this, it arrives to focus the attention on the Higgs mechanism that is at the base of spontaneous symmetry breaking in the standard model), our model introduces a 3D background, a 3D physical vacuum without the need to postulate the consideration of other processes (involving other more fundamental particles): it is based only on the Planckian metric which leads directly to the definition of a Planck energy density as the maximum density that can be possibly sustained in space, and in order to obtain that result it invokes only well-known and simple relations deriving from relativity and quantum mechanics. However, as we will show in the next chapters, both models have at the basis the same underlying consideration: an "effective" curvature might originate from the same physical flat-space vacuum, from the same physical flat-space background.

The quantum vacuum defined by equation (18) identifies a 3D Euclid space as a preferred fundamental arena, which is quantitatively defined by Galilean transformations for the three spatial dimensions

$$\begin{aligned} X' &= X - v \cdot \tau \\ Y' &= Y \\ Z' &= Z \end{aligned} \tag{19}$$

and Selleri's transformation

$$\tau' = \sqrt{1 - \frac{v^2}{c^2}} \cdot \tau \tag{20}$$

for the rate of clocks. In equations (19) and (20) $v$ is the velocity of the moving observer O' of the inertial frame o' measured by the stationary observer O and $\tau$ is the proper time of the observer O of the rest frame o, namely the speed of clock of the observer O (here, as usual, the origin of o', observed from o, is seen to move with velocity $v$ parallel to the $X$ axis). The transformation of the speed of clocks (20) does not contain the space variable. According to the transformations (19) and (20), the temporal coordinate has thus a different ontological status with respect to the spatial coordinates. It is the motion relative to the rest frame of the Euclidean space associated with the quantum vacuum energy density (18) that influences the clocks' running. More precisely, rest mass and relativistic energy of a free particle or body can be considered as phenomena which arise from the diminishing of the quantum vacuum energy density [48]. Furthermore, in this model time exists only as a mathematical parameter measuring the sequential order of material changes occurring in physical states of a system, and the crucial role is played by 3D physical space characterized by the quantum vacuum energy density originating inertial mass [49]. The duration of material changes satisfying the standard Lorentz transformation for the temporal coordinate is a proper, physical scaling function which emerges from the more fundamental numerical order defined by equation

$$t = \tau \left(1 - \frac{v^2}{c^2}\right) + \frac{vX}{c^2} \tag{21}$$

and determines itself a the re-scaling factor of the distance in the first spatial coordinate determined by the material motion of the form

$$\delta = \sqrt{\frac{1}{\tau}\left(t - \frac{vX}{c^2}\right)} \tag{22}$$

which yields just a re-scaling of the position measured by the moving observer expressed by the standard Lorentz transformation for the first spatial coordinate [50].

While in Consoli's model, gravity is determined by the density fluctuations of the elementary spinless quanta (and of their scattering length) characterizing the Bose condensate, in our model gravity is explained without the need to postulate the presence of other hypothetical more fundamental particles: in our model, the origin of gravitational phenomena lies simply in the changes of the energy density of a three-dimensional background space. In our model, the Planck energy density can be considered as the ground state of the same physical flat-space background; the appearance of material objects and subatomic particles correspond to changes of the quantum vacuum energy density and thus can be considered as the excited states of the same physical flat-space background. The excited state of the 3D quantum vacuum corresponding to the appearance of a material particle of mass $m$ is defined by a quantum vacuum energy density (in the centre of this particle) given by equation

$$\rho = \rho_{pE} - \frac{m \cdot c^2}{V} \tag{23}$$

where

$$V = \frac{4}{3} \cdot \pi \cdot R^3 \tag{24}$$

is the volume of the particle (intended as a sphere) and $R$ is the radius of the massive particle [51]. Gravity is a phenomenon determined by the changes of the quantum vacuum energy density and thus by the excited states of the flat-space background. The appearance of a material particle of mass $m$ corresponds to an excited state of the 3D flat-space background defined by a change of energy density (with respect to the ground state) given by equation

$$\rho_{pE} - \rho \equiv \Delta\rho = \frac{m \cdot c^2}{V} \tag{25}$$

In other words, each material particle endowed with mass is produced by a change of the quantum vacuum energy density on the basis of equation

$$m = \frac{V \cdot \Delta\rho}{c^2} \tag{26}$$

On the basis of equation (26), the property of mass derives from a change of the quantum vacuum energy density. Equation (26) indicates that the mass of a given massive body or particle is the result of the interaction of that body or particle with space, in particular it corresponds to an opportune change of the quantum vacuum energy density, which can be considered as the fundamental property of the universal space. Each body or particle diminishes quantum vacuum energy density. We measure this diminishing of quantum vacuum energy density as “inertial mass” and as “gravitational mass”.

The foundational ideas of our model (illustrated in this chapter) can be synthesized in the following postulates (which can be considered as the fundamental postulates of our three-dimensional quantum vacuum energy density model):

1. The medium of space is an isotropic, granular, three-dimensional (3D) "quantum vacuum" constituted by energetic packets having the size of Planck's volume and whose most universal physical property is the energy density.
2. In the free space, without the presence of massive particles, the quantum vacuum energy density is at its maximum and is given by equation (17) (or the equivalent equation (18)) which defines the so-called "ground state" of the 3D quantum vacuum.
3. In the three-dimensional space, the appearance of matter derives from an opportune excited state of the 3D quantum vacuum corresponding to an opportune change of the quantum vacuum energy density. The excited state of the quantum vacuum corresponding to the appearance of a material particle of mass $m$ is defined (in the centre of that particle) by the energy density (23) (and by the change of the energy density (25), with respect to the ground state), depending on the amount of mass $m$ and the volume $V$ of the particle.

In the following chapters, we will show that this model will allows us to obtain results which are compatible with general theory of relativity and will introduce the interesting perspective to see gravity and quantum behaviour of matter as two aspects of the same coin, inside a same unifying picture.

## 4 Curvature of space-time and dark energy in the three-dimensional quantum vacuum energy density model

The Planck energy density (18) is usually considered as the origin of the dark energy and thus of a cosmological constant, if the dark energy is supposed to be owed to an interplay between quantum mechanics and gravity. However the observations are compatible with a dark energy

$$\rho_{DE} \cong 10^{-26} Kg/m^3 \tag{27}$$

and thus equations (18) and (27) give rise to the so-called “cosmological constant problem” because the dark energy (27) is 123 orders of magnitude lower than (18).

In the commonly accepted theory of the universe, dark energy is a form of energy that opposes to gravity generating the expansion of the universe. Its real origin is still a mystery: one explanation is that it could be a property of space itself that increases with the space expansion (Eianl11.9cre9f limit of an underlying "microscopic" structure, of a 3D quantum vacuum condensate whose most universal physical property is its energy density and whose quantum evolution can be seen as the coherent superposition of virtual fine–grained histories. In this picture, space itself can be seen as the hydrodynamic state of a condensate quantum gas, like a Bose–Einstein condensate, in which bosons experience a common collective coherent quantum behaviour described by a macroscopic wave function. In the low energy – long wavelength limit space emerges from the microscopic 3D quantum vacuum, in which the metric and its perturbation correspond to collective variables and collective excitations. A fine–grained history which describes the 3D quantum vacuum is defined by the value of a field $\Phi(x)$ at the point $x$and has quantum amplitude $\Psi[\Phi] = e^{iS[\Phi]}$ , where $S$ is the classical action corresponding to the considered history. The quantum amplitude for a coarse–grained history is defined by:

$$\Psi[\omega] = \int D_F \Phi e^{iS} \omega[\Phi] \tag{30}$$

where

$$D_F[\Phi_A, \Phi_B] \approx \Psi[\Phi_A]\Psi[\Phi_B]^* \approx e^{i(S[\Phi_A] - S[\Phi_B])} \tag{31}$$

is the "decoherence" functional measuring the quantum interference between two virtual histories A and B, $\omega$ can be considered as a "filter" function that selects which fine–grained histories are associated to the same superposition with their relative phases. The decoherence functional for a couple of coarse–grained histories is then:

$$D_F[\omega_A, \omega_B] = \int D_F \Phi_A D_F \Phi_B e^{i(S[\Phi_A] - S[\Phi_B])} \omega[\Phi_A]\omega[\Phi_B]^* \tag{32}$$

in which the histories $\Phi_A$ and $\Phi_B$ assume the same value at a given time instant, where decoherence indicates that the different histories contributing to the full quantum evolution can exist individually, are characterized by quantum amplitude and that the system undergoes an information and predictability degradation [56]. By applying the formalism (32) to hydrodynamics variables [57], Einstein's stress-energy tensor can be expressed through the following operator:

$$\hat{T}_{\mu\nu}(x_A, x_B) = \Gamma_{\mu\nu}\Phi(x_A)\Phi(x_B) \tag{33}$$

where $\Gamma_{\mu\nu}$ is a generic field operator defined at two points that leads to the "conservation law"

$$\hat{T}_{\mu\nu}^{;\nu} = 0 \tag{34}$$

meaning that the decohered quantities, showing a classical behaviour, are the conserved ones.

Moreover, in analogy with Consoli's model, taking into consideration the long wavelength-low energy modes, in the model of the authors the microscopic structure of the underlying background (from which the collective variables derive) can be characterized by the density fluctuations of the vacuum described by the underlying field $\Phi(x)$ which define the fine-grained histories (that in weak gravitational fields, on a coarse-grained scale, is identified with the Newtonian potential

$$\Phi \approx U_N = -G_N \sum_i \frac{M_i}{|r - r_i|} \tag{35}$$

namely one can write

$$g_{\mu\nu} = g_{\mu\nu}\left[\Phi(x)\right] \tag{36}$$

The quantum microscopic structure of the underlying background generating gravity can be characterized by considering the quantum uncertainty principle [58] and the hypotheses of space-time discreteness at the Planck scale. In particular, by applying the discreteness hypothesis of Ng's model, namely the fact that we cannot make $\Delta x$ smaller than the elementary length (7)[1] to Heisenberg's uncertainty relation for the position $\Delta x$ and momentum $\Delta p$

$$\Delta x \geq \frac{\hbar}{2\Delta p} \tag{37}$$

one obtains that, if $\Delta p$ increases, the expression of $\Delta x$ as a function of $\Delta p$ must contain a term directly proportional to $\Delta p$ that counterbalances the term proportional to $(\Delta p)^{-1}$. By following [59], one obtains the following "generalized" version of the uncertainty principle in a discrete space-time:

$$\Delta x \geq \frac{\hbar}{2\Delta p} + \frac{\Delta p}{2\hbar}\left(2\pi^2/3\right)^{2/3} l^{2/3} l_P^{4/3} \tag{38}$$

(38).

By a similar reasoning one can obtain the corresponding version of (38) for time uncertainty as:

$$\Delta t \geq \frac{\hbar}{2\Delta E} + \frac{\Delta E T_0^2}{2\hbar} \tag{39}$$

[1] An analogous limitation holds in time.

where $\Delta E$ is the energy uncertainty and $T_0 = \frac{1}{c}\left(2\pi^2/3\right)^{1/3} l^{1/3} l_P^{2/3}$ is the elementary time. The new terms appearing in equations (38) and (39) indeed represent the "intrinsic" uncertainty of space-time due to the presence of matter of mass (26). As a consequence, the energy $E \approx pc$ contained in a region of size $L$ and deriving from matter of mass (26) modifies the extension of this region of an amount:

$$\Delta L \cong \frac{\left(2\pi^2/3\right)^{1/3} l^{1/3} l_p^{2/3} T_0 E}{2\hbar} \tag{40}$$

On the basis of equation (40), the curvature of space-time can be related to the presence of energy and momentum in it. In other words, in the approach here suggested, one can say that the changes of the quantum vacuum energy density associated to the presence of matter of mass (26) correspond to an underlying microscopic background geometry defined by equation (40).

The expectation value of the stress-energy tensor operator of the quantum fields (33) at any point leads to changes of the 3D quantum vacuum energy density. In order to have the correct Friedmann-Robertson-Walker metric, this assumption means that

$$\langle\Psi| \hat{T}_4^4 |\Psi\rangle = \frac{\Delta\rho_{qvE}}{c^2}; \quad \langle\Psi| \hat{T}_\mu^\nu |\Psi\rangle \approx 0 \text{for} \mu\nu \neq 00 \tag{41}$$

$\Psi$ being the quantum state of the universe corresponding to the value of the field $\Phi(x)$ defining a given fine-grained history. This suggests to express the stress-energy tensor (33) corresponding to the quantum vacuum fluctuations as

$$\hat{T}_{\mu\nu}^{vac} \equiv \hat{T}_{\mu\nu} - \langle\Psi| \hat{T}_{\mu\nu} |\Psi\rangle \hat{I} \tag{42}$$

where $\hat{I}$ is the identity operator. The existence of quantum vacuum fluctuations imply that, despite the expectation of $\hat{T}_{\mu\nu}^{vac}$ is zero by definition, one has

$$\langle\Psi| \hat{T}_{\mu\nu}^{vac}(x)\, \hat{T}_{\lambda\sigma}^{vac}(y) |\Psi\rangle \neq 0 \tag{43}$$

in general.

Now, in the approach developed by the authors in [55], in order to derivate the curvature of space-time associated with a dark energy density as a mathematical value of the changes of the 3D quantum vacuum energy density (whose underlying microscopic structure is characterized by a geometry expressed by equations (38)-(40)), taking account of Santos' results, we assume that the dark energy density (27) is associated with opportune fluctuations $\Delta\rho_{qvE}^{DE}$ of the quantum vacuum energy density on the basis of equation

$$\rho_{DE} \cong 70\frac{G}{4\pi}\left(\frac{V}{c^2}\Delta\rho_{qvE}^{DE}\right)^2\frac{1}{l}\cdot\frac{1}{l^3} \tag{44}$$

*where*

$$l = \frac{\hbar}{\left(\frac{V}{c^2}\Delta\rho_{qvE}^{DE}\right)c} \tag{45}$$

(45).

In other words, this means that the integral of the two-point correlation function is assimilated to opportune fluctuations of the quantum vacuum energy density on the basis of equation

$$\left(\frac{V}{c^2}\Delta\rho_{qvE}^{DE}\right)^2\frac{1}{l}\cdot\frac{1}{l^3} = 4\pi\int_0^\infty C\left(r_{12}\right)r_{12}dr_{12} \tag{46}$$

In order to derive the correct Friedmann-Robertson-Walker metric, in [55] we used a metric of the quantum vacuum defined by relation

$$d\hat{s}^2 = \hat{g}_{\mu\nu}dx^\mu dx^\nu \tag{47}$$

where the coe□cients (in polar coordinates) are

$$\begin{gathered} hatg_{00} = -1 + \hat{h}_{00}, \quad \hat{g}_{11} = 1 + \hat{h}_{11}, \quad \hat{g}_{11} = 1 + \hat{h}_{11}, \\ \hat{g}_{33} = r^2\sin^2\vartheta\left(1 + \hat{h}_{33}\right), \quad \hat{g}_{\mu\nu} = \hat{h}_{\mu\nu}\mathrm{for}\mu \neq \nu \end{gathered} \tag{48}$$

where multiplication of every term times the unit operator is implicit and, at the order $O\left(r^2\right)$, one has

$$\begin{gathered} \left\langle\hat{h}_{\mu\nu}\right\rangle = 0 \text{ except } \left\langle\hat{h}_{00}\right\rangle = \frac{8\pi G}{3}\left(\frac{\Delta\rho_{qvE}}{c^2} + \frac{35Gc^2}{2\pi\hbar^4 V}\left(\frac{V}{c^2}\Delta\rho_{qvE}^{DE}\right)^6\right)r^2 \\ \text{and}\left\langle\hat{h}_{11}\right\rangle = \frac{8\pi G}{3}\left(-\frac{\Delta\rho_{qvE}}{2c^2} + \frac{35Gc^2}{2\pi\hbar^4 V}\left(\frac{V}{c^2}\Delta\rho_{qvE}^{DE}\right)^6\right)r^2 \end{gathered} \tag{49}$$

where $\left\langle \hat{h}_{\mu\nu} \right\rangle$ stands for $\langle\Psi|\,\hat{h}_{\mu\nu}\,|\Psi\rangle$ (and the fluctuations of the quantum vacuum $\Delta\rho_{qvE} = \rho_{pE} - \rho_{qvE}$ correspond to an underlying microscopic geometry defined by equations (38)-(40)).

By starting from the quantized metric (47) whose coe□cients are defined by relations (48) and (49), in the approximation of the second order in the (small) tensor $\hat{h}_{\mu\nu}$, it is possible to derive the components of quantum Einstein equations of the form

$$\hat{G}_{\mu\nu} = \frac{8\pi G}{c^4}\hat{T}_{\mu\nu} \tag{50}$$

(where the quantum Einstein tensor operator $\hat{G}_{\mu\nu}$ is expressed in terms of the operators

$\hat{h}_{\mu\nu}$) on the basis of the assumption that they are similar to the classical counterparts. In particular, the expectation value of the (operator) metric parameter $\hat{h}_{11}$ may be written in the form

$$\langle\Psi|\,\hat{h}_{11}\,|\Psi\rangle = \langle\Psi|\,\hat{h}_{11}\,|\Psi\rangle_{mat} + \langle\Psi|\,\hat{h}_{11}\,|\Psi\rangle_{vac} \tag{51}$$

namely it is the sum of two expressions, one containing the matter density produced by the changes of the quantum vacuum energy density, and the other indicating the vacuum density fluctuations. Taking into account that according to Santos' results, the vacuum contribution appearing in equation (41), to order $G^2$, is

$$\langle\Psi|\,\hat{h}_{11}\,|\Psi\rangle_{vac} \cong 600G^2r^2\int_0^\infty C\,(s)\,sds \tag{52}$$

$r$being a distance which is estimated to fulfil $r/s \approx 10^{40}$, the vacuum contribution may be expressed as

$$\langle\Psi|\,\hat{h}_{11}\,|\Psi\rangle_{vac} \cong \quad 150\frac{1}{\pi}G^2r^2\left(\frac{V}{c^2}\Delta\rho_{qvE}^{DE}\right)^2\frac{1}{l}\cdot\frac{1}{l^3} \tag{53}$$

Therefore, the total expectation value (51) becomes, to order $r^2$

$$\langle\Psi|\,\hat{h}_{11}\,|\Psi\rangle \cong \frac{8\pi G\Delta\rho_{qvE}}{3c^2}r^2 + 150\frac{1}{\pi}G^2r^2\left(\frac{V}{c^2}\Delta\rho_{qvE}^{DE}\right)^2\frac{1}{l}\cdot\frac{1}{l^3} \tag{54}$$

Hence, comparison with the Friedmann equations, taking account of relations (44) and (52), leads to the following equation

$$\rho_{DE}c^2 \cong \frac{35G}{2\pi V}\left(\frac{V}{c^2}\Delta\rho_{qvE}^{DE}\right)^2 \frac{1}{l}\cdot\frac{1}{l^3} \tag{55}$$

namely

$$\rho_{DE} \cong \frac{35Gc^2}{2\pi\hbar^4 V}\left(\frac{V}{c^2}\Delta\rho_{qvE}^{DE}\right)^6 \tag{56}$$

which states the equivalence of the curvature of space-time produced by the changes of the quantum vacuum energy density and the one determined by a constant dark energy density. This means that in the approach based on equations (37)-(56), the changes and fluctuations of the quantum vacuum energy density generate a curvature of space-time similar to the curvature produced by a "dark energy" density. In synthesis, one can say that the curvature of space-time may be considered as a mathematical value which emerges from the quantized metric (47) of the quantum vacuum condensate whose coefficients are defined by equations (48) and (49) (and whose underlying microscopic geometry is described by equations (38)-(40)) and thus from the changes and fluctuations of the quantum vacuum energy density.

So, gravitational interaction emerges as an effect of the changes of the 3D quantum vacuum energy density. The excited state of the 3D quantum vacuum characterized by a change of the energy density given by (25) can be also described by a gravitational energy density given by equation

$$\rho_{grav}c^2 = -G\left(\frac{V}{c^2}\Delta\rho_{qvE} + \frac{35Gc^2}{2\pi\hbar^4}\left(\frac{V}{c^2}\Delta\rho_{qvE}^{DE}\right)^6\right)^2 \frac{1}{l}\cdot\frac{1}{l^3} \tag{57}$$

In this approach, the variable quantum vacuum energy density (given by equation (23)) and its corresponding variable gravitational energy density (given by equation (57)) can be considered the fundamental elements that determine the gravitational interaction between material objects. Considering the 3D quantum vacuum characterized by a ground state defined by the energy density (18) as the fundamental arena of the universe, a concept of *curvature/density of space* can be introduced which means that presence of mass (determined by an opportune excited state of the 3D quantum vacuum corresponding to a given change

(25) of its energy density) decreases the quantum vacuum energy density and increases its effective curvature: gravity is carried by the effective curvature caused by a variable quantum vacuum energy density. It is interesting to remark that, inside this model, gravitational interaction is an immediate, direct phenomenon: the 3D quantum vacuum (through its fundamental quantity represented by the variable gravitational energy density of the quantum vacuum (57)) acts as a direct medium of gravity. No movement of particle-wave is needed for its acting: gravity is transmitted directly by the variable quantum vacuum energy density, by means of its corresponding variable gravitational energy density characterizing the region between the material objects under consideration. Gravity is the result of the geometrical shape of the quantum vacuum energy density. The gravitational energy density (57) associated with an opportune excited state of the 3D quantum vacuum defined by the energy density (23) (and thus by a change of the energy density (25) with respect the ground state (18)) acts as a direct medium of gravity, in the sense that the two-point correlation function (29) satisfies relation

$$4\pi G\int_0^\infty C\left(r_{12}\right)r_{12}dr_{12}=G\left(\frac{V}{c^2}\Delta\rho_{qvE}+\frac{35Gc^2}{2\pi\hbar^4}\left(\frac{V}{c^2}\Delta\rho_{qvE}^{DE}\right)^6\right)^2\frac{1}{l}\cdot\frac{1}{l^3} \tag{58}$$

namely

$$4\pi\int_0^\infty C\left(r_{12}\right)r_{12}dr_{12}=\left(\frac{V}{c^2}\Delta\rho_{qvE}+\frac{35Gc^2}{2\pi\hbar^4}\left(\frac{V}{c^2}\Delta\rho_{qvE}^{DE}\right)^6\right)^2\frac{1}{l}\cdot\frac{1}{l^3} \tag{59}$$

Since in the second member time does not appear, equation (59) physically means that gravity-space is a direct phenomenon: gravity is transmitted directly by the changes of the 3D quantum vacuum energy density defining its excited states, no time is needed for the transfer of the gravitational energy density of quantum vacuum from one point to another, in other words the variable gravitational energy density of quantum vacuum acts as a direct medium for the transmission of gravitation.

Finally, let us see how the curvature of space-time corresponding to the changes and fluctuations of the quantum vacuum energy density – which characterize a given excited state of the 3D quantum vacuum – acts on a test particle of mass $m_0$, in other words how the motion of a material object in a background characterized by changes of its energy density can be treated mathematically. When a material object, determined by an excited state of the 3D quantum vacuum and thus by a given diminishing of the quantum vacuum energy density, moves, this diminishing of the quantum vacuum energy density causes a shadowing of the gravitational space – similar to the idea of the polarizability of the vacuum in the vicinity of a mass (or other mass-energy concentrations) introduced by Putho□'s polarizable model of gravitation – which determines the motion of other material objects present in the region under consideration. In [55] we have made the assumption that the shadowing of the 3D quantum vacuum can be expressed by equation

$$D = \kappa \varepsilon_0 E_g \tag{60}$$

where $\kappa$ is a factor which represents the relatively small amount of the altered permittivity of the free space (with respect to the situation in which the energy density of the quantum vacuum is given by equation (18)) and

$$E_g = -H_{eg}\left(\frac{V}{c^2}\Delta\rho_{qvE} + \frac{35Gc^2}{2\pi\hbar^4}\left(\frac{V}{c^2}\Delta\rho_{qvE}^{DE}\right)^6\right)\frac{1}{r^2}\hat{r} \tag{61}$$

is the gravitostatic field determined by both density of matter and density of dark energy (here $H_{eg} = \frac{G}{c^2}$ is the basic gravitodynamic constant). The total lagrangian density. In analogy with Putho□'s polarizable vacuum model of gravitation [4], variation of the action functional given by relation

$$\begin{aligned} L_d = &-\left(\frac{m_0c^2}{\sqrt{K}}\sqrt{1-\left(\frac{v}{c/K}\right)^2} + q\Phi - qA\cdot v\right)\delta^3\left(r-r_0\right) \\ &-\frac{1}{2}\left(\frac{B_g^2}{K\mu_0} + K\varepsilon_0 E_g^2\right) - \frac{\lambda}{K^2}\left(\left(\nabla K\right)^2 + \frac{1}{\left(c/K\right)^2}\left(\frac{\partial K}{\partial t}\right)^2\right) \end{aligned} \tag{62}$$

(where $\left(\Phi, A\right)$ are the gravitational potentials, $B$ is the gravitomagnetic field defined by

$$B_g = H_{eg}\frac{J}{r^3} \tag{63}$$

$J = L + S,\quad L = r\times\left(\frac{V}{c^2}\Delta\rho_{qvE} + \frac{35Gc^2}{2\pi\hbar^4}\left(\frac{V}{c^2}\Delta\rho_{qvE}^{DE}\right)^6\right)v,$ $S$ being the spin angular momentum of the material object determined by the diminishing of the quantum vacuum energy density under consideration and $\lambda = \frac{c^4}{32\pi G}$) with respect to the test particle variables leads to the following equation of motion of a test material object of mass $m_0$ in the polarized 3D quantum vacuum:

$$\frac{d}{dt}\left[\frac{\left(m_0\kappa^{3/2}\right)v}{\sqrt{1-\left(\frac{v}{c/\kappa}\right)^2}}\right] = m_0\left(c^2E_g + v\times B_g\right) + \frac{m_0c^2}{\kappa}\cdot\frac{1+\left(\frac{v}{c/\kappa}\right)^2}{2\sqrt{1-\left(\frac{v}{c/\kappa}\right)^2}}\cdot\frac{\nabla\kappa}{\kappa} \tag{64}$$

Equation (64) shows that there are two forces acting onto the test particle of mass $m_0$: the Lorentz force due to the quantum vacuum energy density surrounding the object and a second term representing the dielectric force proportional to the gradient of the shadowing of the quantum vacuum (60). The importance of this second term lies in the fact that thanks to it one can account for the gravitational potential, either in Newtonian or general relativistic form. In agreement with general relativity, with $m_0 \to 0$ and $v \to \frac{c}{\kappa}$, as would be the case for a photon, the deflection of the trajectory is twice as the deflection of a slow moving massive particle.

Variation of the action functional with regard to the $\kappa$ variable leads to the expression of the generation of the shadowing of space within general relativity, owed to the presence of both matter and fields:

$$\nabla^2\sqrt{\kappa} - \frac{1}{(c/\kappa)^2}\cdot\frac{\partial^2\sqrt{\kappa}}{\partial t^2} = \frac{-\kappa}{4\lambda}\left[P(\kappa)+Q(\kappa)+R(\kappa)\right] \tag{65}$$

Here $P(\kappa)$ represents the change in space shadowing by the density of matter associated with the object of mass $m_0$, with the vector $r$ as the distance from the system mass centre:

$$P(K) = \frac{m_0c^2}{\sqrt{K}}\cdot\frac{1+\left(\frac{v}{c/K}\right)^2}{\sqrt{1-\left(\frac{v}{c/K}\right)^2}}\cdot\delta^3(r-r_0) \tag{66}$$

$Q(\kappa)$ is the change caused by the energy density of the fields (61) and (63) determined by the diminishing of the quantum vacuum energy density:

$$Q(\kappa) = \frac{1}{2}\left(\frac{B_g^2}{\kappa\mu_0} + \kappa\varepsilon_0E_g^2\right) \tag{67}$$

$R(\kappa)$ is the change caused by the shadowing of the quantum vacuum energy density itself:

$$R(\kappa) = -\frac{\lambda}{\kappa^2}\left((\nabla\kappa)^2 + \frac{1}{(c/\kappa)^2}\left(\frac{\partial\kappa}{\partial t}\right)^2\right) \tag{68}$$

In the case of a static gravity field of a spherical mass distribution (a planet or a star), the solution of equation (65) has a simple exponential form:

$$\sqrt{\kappa} = e^{GM/rc^2} \tag{69}$$

where $M = \frac{V\Delta\rho_{qvE}}{c^2}$. The solution (69) can be approximated by expanding it into a series:

$$\kappa = e^{2GM/rc^2} = 1 + \frac{2GM}{rc^2} + \frac{1}{2}\left(\frac{2GM}{rc^2}\right)^2 + ... \tag{70}$$

This solution reproduces (to the appropriate order) the usual general-relativistic Schwarzschild metric predictions in the weak field limit conditions (i.e. Solar system).

According to equation (67) also a photon will add a contribution to the effective curvature of space associated with the fields (61) and (63). This result turns out to be also in accordance with general theory of relativity, where both mass and energy cause the curvature of space.

Moreover, the obtained solution (69) (or (70)) concerning the factor $\kappa$ measuring the polarizability of the quantum vacuum allows us to analyze the gravitational red shift characteristic of general relativity, and find a more detailed formula in order to evaluate the frequency shift of the photon emitted by an atom on the surface of a star of mass $M$ and radius $R$. In this regard, just like in Puthoff's model, the photon detected far away from the star will appear red shifted by the following amount:

$$\frac{\Delta\omega}{\omega_0} = \frac{\omega - \omega_0}{\omega_0} \approx -\frac{GM}{Rc^2} \tag{71}$$

where $\frac{GM}{Rc^2} << 1$.

Finally, the same result allows us to derive the amount of the bending of light rays from a distant star passing near a massive body, like in the classic general relativity test performed by Eddington's expedition during the solar eclipse in May 1919. The light ray coming from a distant star, will experience a gradual slowing of wavefront velocity coming towards the Sun, and a gradual increasing velocity in leaving Sun's gravity field. This is seen from Earth as an apparent shift of the position of the star close to Sun's disk edge in the outward direction. The total bending angle may be calculated by approximating the variable velocity of light to the first order term of the series expansion (70) of $\kappa$:

$$v = \frac{c}{\kappa} \approx \frac{c}{1 + \frac{2GM}{rc^2}} \approx c\left(1 - \frac{2GM}{rc^2}\right) \tag{72}$$

In this relation the radius-vector $r$ indicates the distance of the wavefront from the centre of the Sun as it travels by from $-\infty$ to $+\infty$, with the minimum distance of $R+\delta$ where $R$ is Sun's radius and $\delta$ is the minimum distance from Sun's surface. By assigning $z$ to the distance of the wavefront along the line of sight (perpendicular to $R+\delta$), the radius-vector becomes $r = \sqrt{(R+\delta)^2 + z^2}$, so equation (72) can be written as:

$$v \approx c\left(1 - \frac{2GM}{c^2} \cdot \frac{1}{\sqrt{(R+\delta)^2 + z^2}}\right) \tag{73}$$

As the wavefront travels a distance $dz = v\,dt$, the change of the velocity (73) along the path of light results an accumulated tilt angle of:

$$\phi \approx \Delta z/\delta \approx \frac{2GM}{c^2} \cdot \frac{R}{(R^2 + z^2)^{3/2}} dz \tag{74}$$

Integration of equation (74) over the entire path $-\infty < z < +\infty$ yields:

$$\phi \approx \frac{4\pi GM}{Rc^2} \tag{75}$$

By inserting $G = 6,672 \cdot 10^{-11} Nm^2 Kg^{-2}$, $M = 1,9891 \cdot 10^{30} Kg$, and $R = 6,96 \cdot 10^8 m$, we obtain $\phi = 1,75$ arc-seconds, which is exactly the value predicted by Einstein's general theory of relativity in 1915, and experimentally verified by Eddington in 1919 (between 1.2 and 1.9 arc-seconds, mainly because of the imperfect optics of the portable telescopes used).

In synthesis, in this model, one can say that the equation of motion (64) for a test material object in the 3D quantum vacuum, characterized by a decreasing of the energy density (associated with a given excited state of the 3D quantum vacuum) on one hand, and the equation (65) describing the shadowing of the background space produced by the presence of matter and fields on the other hand, allow us to obtain results that are coherent and compatible with general theory of relativity. The perspective is thus opened that the curvature of space of general relativity can be associated with the diminishing of the quantum vacuum energy density (corresponding to a given excited state of the quantum vacuum and determining the presence of material objects), and that the material objects follow the geodesic paths within the shadowed gravitational space (determined just by the change of the quantum vacuum energy density and thus associated with an excited state of the quantum vacuum). Moreover, as regards the equations of motion (64) and (65), it is important to emphasize that, according to this approach, the modification of the quantum vacuum energy density (corresponding to a given excited state of the quantum vacuum and determining both the matter density and dark energy density) as well as the action of the shadowed quantum vacuum on another material object are phenomena directly determined by the fields (63), (66), (67) and (68). This means that no time is needed to transmit the information from a material object to the surrounding region in order to shadow the gravitational space because the change of the quantum vacuum energy density is already there as it is associated with the fields (63), (66), (67) and (68) (what propagates from point to point is just the effective consequences of this change); and, on the other hand, no time is needed to transmit the information from the shadowed space to another material object in order to cause its movement.

## 5 About the interpretation of the quantum behaviour of matter in the three-dimensional quantum vacuum energy density model

In the 3D quantum vacuum energy density model, the quantum behaviour of matter derives directly from the features of the excited states of the quantum vacuum. The changes of the quantum vacuum energy density corresponding to the matter of mass (25) determine a gravitostatic field and a gravitomagnetic field given by relations

$$E_g = -H_{eg}\left(\frac{V}{c^2}\Delta\rho_{qvE}\right)\frac{1}{r^2}\hat{r} \tag{76}$$

$$B_g = H_{eg}\frac{J}{r^3} \tag{77}$$

where

$$J = L + S, \quad L = r \times \left(\frac{V}{c^2}\Delta\rho_{qvE}\right) v \tag{78}$$

The fields (76) and (77) have the physical form of a wave vector and a frequency respectively. The nature of the fields (76) and (77) leads thus us to introduce the following wave function of the excited states of the quantum vacuum

$$\psi_S = A\exp\left[2\pi i\left(H_{eg}\frac{V}{c^2}\Delta\rho\frac{1}{r^2}\hat{r}\cdot r_0 - H_{eg}\frac{J}{r^3}t + \phi_0\right)\right] \tag{79}$$

where the amplitude $A$ is a function of the generic point $(x, y, z)$ of space. $A$ will depend in general on the quantum vacuum energy density associated with the excited state under consideration and on the speed of the particle determined by this excited state.

Inside our quantum vacuum energy density model, the wave function (79) of the quantum vacuum may be considered as physically real in the sense that, in the particular case in which the excited state of the quantum vacuum determines the presence of a particle with spin 0, where it assumes the form

$$\psi_S = A\exp\left[2\pi i\left(H_{eg}\frac{V}{c^2}\Delta\rho\frac{1}{r^2}\hat{r}\cdot r_0 - H_{eg}r \times \frac{V}{c^2}\Delta\rho v\frac{1}{r^3}t + \phi_0\right)\right] \tag{80}$$

it satisfies the fundamental and general (non linear) generalized Klein-Gordon equation for the quantum vacuum energy density

$$\nabla^2\psi_S - \frac{1}{c^2}\frac{\partial^2\psi_S}{\partial t^2} = H_{eg}\frac{\frac{V}{c^2}\Delta\rho}{r^2}\psi_S \tag{81}$$

On the basis of equation (81) it becomes permissible the following re-reading of the behaviour of spinless subatomic particles: spinless subatomic particles satisfying the standard Klein-Gordon equation of quantum mechanics derive from more fundamental excited states of the quantum vacuum described by wave functions of the form (80) which satisfy the more fundamental generalized Klein-Gordon equation for the quantum vacuum energy density (81).

As regards this equation, the non-linearity leads to obvious mathematical di□culties (e.g., the quantization is a complex issue because there is not an exact superposition principle [60]) but on the other side, it clearly puts this entire picture in a promising dynamic perspective. In this regard, one can assume that the properties of the universe are principally nonlinear and that superposition is an approximate principle, but not precise one[2]; electromagnetic and gravitational forces cannot act on infinite distance with the dependence $\frac{1}{r^2}$, it is an approximate expression only.

In the quantum domain, for the time-independent case equation (81) assumes the following form

$$\nabla^2 \psi_S = H_{eg} \frac{\frac{V}{c^2}\Delta\rho}{r^2} \psi_S \tag{82}$$

whose solution is exponential and still non linear.

The solutions of equation (82) depend on the $r$-domain. Defining $r_g = H_{eg}\frac{V}{c^2}\Delta\rho$ we have the following results:

1. For $r > r_g$ (case of weak fields, such as near planets), since $(r_g/r)^2 \approx 0$, equation (82) becomes the well-known Laplace equation

$$\nabla^2 \psi_S = 0 \tag{83}$$

   whose domain is linear and where superposition is valid. This domain could be considered as a classical vector gravity. This result allows us to draw the following fundamental conclusion: linear vector gravitation can be considered an approximation of the most general non linear quantum- vacuum energy density approach here suggested in the particular case $r > r_g$.

2. For $r < r_g$, we have a totally non-classical domain (the so-called "black hole" inner space) with non linear phenomena. In this domain the non linear wave equation is

$$\nabla^2 \psi_S = \left( \frac{H_{eg}\frac{V}{c^2}\Delta\rho}{r^2} \right)^2 \psi_S \tag{84}$$

[2]As we know, superposition holds in cases where the interaction cross-section between particles is very low, such as between photons in the ordinary/default energy density. At energy densities where the interaction cross-section becomes considerable, superposition must be replaced by the effective probability of interaction.

3. For $r = r_g$, a strong field domain is reached. In this case, from the de Broglie-like condition for circular orbits $2\pi r = n\lambda_S$, assuming $\lambda_H = E_g^{-1}$, we have $r = \frac{2\pi r_g}{n}$ namely $r = \frac{2\pi}{n} H_{eg} \frac{V}{c^2} \Delta\rho$ where $1 \leq n \leq 6$. The so-called "black hole" phenomenon can be taken under examination from this point of view. In this domain the wave equation assumes the form

$$\nabla^2 \psi_S = \left( H_{eg} \frac{V}{c^2} \Delta\rho \right)^{-2} \psi_S \tag{85}$$

with general exponential solutions [61, 62], e.g. for one-dimensional case

$$\psi_S(z) = \psi_1 \exp\left[ -\frac{GV \cdot \Delta\rho z}{c^4} \right] + \psi_2 \exp\left[ \frac{GV \cdot \Delta\rho z}{c^4} \right] \tag{86}$$

The points 1, 2 and 3 listed above show clearly that in the treatment of the excited states of the quantum vacuum, corresponding to a change of the energy density given by equation (23), physical interactions such as weak fields of planets or strong fields close to black holes, derive from the particular characteristics of opportune excited states of the quantum vacuum. The physical fields can be interpreted as real properties of the same flat background, which in turn emerge from a peculiar behaviour of the quantum vacuum energy density, more precisely of the excited states of the quantum vacuum. This can be considered as an important perspective of the 3D quantum vacuum approach introduced in this article.

Moreover, the wave function (80) associated with the excited states of the quantum vacuum (which determine the presence of spinless particles) can be considered as the source of the curvature of space-time: in analogy with F. Shojai's and A. Shojai's model, there is a fundamental connection of this wave function with the curvature of space-time characteristic of general theory of relativity.

In order to demonstrate this fundamental result, before all we write a Bohm's version of the generalized Klein-Gordon equation for the quantum vacuum energy density. By decomposing the real and imaginary parts of equation (81), one obtains a nonlinear quantum Hamilton-Jacoby equation for the wave function of the excited states of the quantum vacuum[3] that, by imposing the requirement that it be Poincarè

[3]In many of the following equations we are going to use the 3+1-dimensional

invariant and have the correct non-relativistic limit, assumes the following form

$$\partial_\mu S \partial^\mu S = \frac{V^2 (\Delta\rho)^2}{c^2\hbar^2} \exp Q \tag{87}$$

and the continuity equation

$$\partial_\mu J^\mu = 0 \tag{88}$$

where

$$Q = \frac{c^2\hbar^2}{V^2 (\Delta\rho)^2} \frac{\left(\nabla^2 - \frac{1}{c^2}\frac{\partial^2}{\partial t^2}\right) A}{A} \tag{89}$$

can be interpreted as the quantum potential associated with the wave function (80) of the excited states of the quantum vacuum,

$$S = \frac{2\pi}{\hbar}\left[\left(H_{eg}\frac{V}{c^2}\Delta\rho\frac{1}{r^2}\hat{r}\cdot r_0 - H_{eg} r \times \frac{V}{c^2}\Delta\rho v \frac{1}{r^3} t + \phi_0\right)\right] \tag{90}$$

is the phase of the wave function (80) of the quantum vacuum, $A$ is the amplitude of this wave function and

$$J^\mu = -\frac{c^2 r^2 A^2}{H_{eg} V \Delta\rho}\partial^\mu S \tag{91}$$

is the current associated with this wave function.

Now, starting from Bohm's version of the generalized Klein-Gordon equation for the quantum vacuum energy density, in analogy with A. Shojai's and F. Shojai's model, the link between the wave function of the excited states of the quantum vacuum (80) (and thus of the quantum potential associated with them) and the curvature of space-time can be shown by changing the ordinary di�erentiating $\partial_\mu$ with the covariant derivative $\nabla_\mu$ and by changing the Lorentz metric with the curved metric $g_{\mu\nu}$. In this way we obtain the equations of motion for a change of the quantum vacuum energy density (which determines the appearance of a particle at spin 0) in a curved background:

$$\tilde{g}_{\mu\nu}\tilde{\nabla}_\mu S \tilde{\nabla}_\nu S = \frac{V^2 (\Delta\rho)^2}{c^2\hbar^2} \tag{92}$$

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Riemann-Minkowski notation (with implicit Einstein sum convention) in order to characterize the quantum vacuum.

$$\tilde{g}_{\mu\nu}\tilde{\nabla}_{\mu}J^{\mu}=0 \tag{93}$$

where $\tilde{\nabla}_{\mu}$ represents the covariant differentiation with respect to the metric

$$\tilde{g}_{\mu\nu}=g_{\mu\nu}\exp Q \tag{94}$$

which is a conformal metric, where

$$Q=\frac{c^2\hbar^2}{V^2\left(\Delta\rho\right)^2}\frac{\left(\nabla^2-\frac{1}{c^2}\frac{\partial^2}{\partial t^2}\right)_g A}{A} \tag{95}$$

is the quantum potential.

From the treatment of the generalized Klein-Gordon equation for the quantum vacuum energy density inside a bohmian framework here provided, one can conclude that the effects of gravity on geometry and the quantum effects on the geometry of space-time are highly coupled and the source of both of them is always the 3D quantum vacuum: the space-time geometry sometimes looks like what we call gravity and sometimes looks like what we understand as quantum behaviours and this derives from the properties of opportune excited states of the 3D quantum vacuum. The presence of the quantum potential associated with the wave function (80) of the 3D quantum vacuum is equivalent to a curved space-time with its metric being given by (94). The curving of space-time can be considered an effect, in a certain sense, of the quantum potential associated with the wave function (80) of the 3D quantum vacuum. In other words, there is a significant link between the wave function (80) of opportune excited states of the quantum vacuum and the curvature of space-time. We have in this way provided a sort of geometrization of the quantum aspects of the changes of the quantum vacuum energy density (in the case in which they determine the presence of a spinless particle).

On the basis of the approach developed in this chapter, we can say that the changes of the quantum vacuum energy density corresponding to opportune excited states of the 3D quantum vacuum determine the curvature of space-time and at the same time the space-time metric is linked with the quantum potential associated with these excited states of the 3D quantum vacuum and which influences the behaviour of the particles determined by these same excited states. The quantum potential associated with the excited states of the quantum vacuum creates itself a curvature which may have a large influence on the classical contribution to the curvature of the space-time. In the approach here suggested, it becomes so permissible the following re-reading of the results obtained by F. Shojai and A. Shojai inside their toy model (mentioned in the introduction): the particles determine the curvature of space-time and at the same time the space-time metric is linked to the quantum potential which influences the behaviour of the particles just as a consequence of the behaviour of the energy density of opportune excited states of the quantum vacuum and of their wave functions. The space-time geometry sometimes looks like what we call gravity and sometimes look like what we understand as quantum behaviour just as a consequence of the features of the excited states of the quantum vacuum: the wave function of the quantum vacuum represents a sort of link between these two elements, which indeed can be seen as two aspects of the same coin, of the same source.

Another important feature of the approach of the wave function of quantum vacuum based on equations (80)-(95) lies in the non-locality. On the basis of the quantum potential (95), which represents the action of the wave function of the excited states of the quantum vacuum on fundamental changes of the quantum vacuum energy density (producing the appearance of spinless particles), this approach is highly non-local. The non-locality characterizing the processes described by the wave function (80) of the quantum vacuum (which determines just the appearance of a spinless particle) can be characterized in a simple and direct way by considering the quantity

$$S_Q = -\frac{1}{2}\ln\rho \tag{96}$$

where $\rho = |\psi(x,t)|^2$ is the probability density associated with the wave function $\psi(x,t)$ of the 3D quantum vacuum. The quantity (96) can be interpreted as a physical entity that describes the degree of order and chaos of the 3D quantum vacuum supporting the density $\rho$ describing the space-temporal distribution of an ensemble of elementary quasi-particles associated with the wave function of the quantum vacuum under consideration and thus can be appropriately defined quantum entropy of the quantum vacuum, or more briefly quantum vacuum entropy. Starting from the quantum vacuum entropy (96), the quantum potential (95) may be conveniently written in the following form

$$Q = -\frac{c^2\hbar^2}{V^2\Delta\rho^2}\left(\nabla_\mu S_Q\right)_g^2 + \frac{c^2\hbar^2}{V^2\Delta\rho^2}\left(\left(\nabla^2 - \frac{1}{c^2}\frac{\partial^2}{\partial t^2}\right)_g S_Q\right) \tag{97}$$

and thus the conformal metric (94) reads as

$$\tilde{g}_{\mu\nu} = g_{\mu\nu} \exp\left[- \frac{c^2\hbar^2}{V^2\Delta\rho^2} \left(\nabla_\mu S_Q\right)_g^2 + \frac{c^2\hbar^2}{V^2\Delta\rho^2} \left(\left(\nabla^2 - \frac{1}{c^2}\frac{\partial^2}{\partial t^2}\right)_g S_Q\right)\right] \tag{98}$$

Equation (97) suggests that the quantum potential of the vacuum for the generalized Klein-Gordon equation is an information channel given by the sum of two quantum correctors linked with the quantum vacuum entropy; and, on the basis of equation (98), the quantum vacuum entropy emerges as the informational line which determines the conformal metric. The link between space-time geometry and quantum behaviour of matter emerges therefore as an effect of the fact that the density of fundamental quasi-particles associated with a given function of the 3D quantum vacuum determines a degree of order and chaos, and thus a deformation of the geometry, of the background of the processes.

Moreover, and this is the crucial point in order to explain in a direct way the non-locality of the approach, starting from the expression (97) of the quantum potential as a sum of two quantum correctors, it is possible to introduce a characteristic quantum length associated with the conformal metric (98) determined by the quantum vacuum entropy:

$$L_{quantum} = \frac{1}{\sqrt{\left(\nabla_\mu S_Q\right)_g^2 - \left(\nabla^2 - \frac{1}{c^2}\frac{\partial^2}{\partial t^2}\right)_g S_Q}} \tag{99}$$

The quantum length (99) measures the strength of quantum effects and, therefore, the modification of the geometry – introduced in a relativistic curved space-time regime – with respect to the Euclidean geometry characteristic of classical physics. Once the quantum length (99) becomes non-negligible the change of the quantum vacuum energy density determining the appearance of a spinless particle goes into a quantum regime where the quantum and gravitational effects are highly related.

On the basis of equation (99), the only case in which one may obtain 0 in the denominator of the quantum length is the classical one, namely corresponding to a quantum potential equal to 0. This means, from the physical point of view, that the quantum length (99) practically measures the range of a quantum informational gradient, in other words the degree of non-locality in a region of the 3D quantum vacuum. The classical limit can be expressed thus by the condition

$$(\nabla_\mu S_Q)_g^2 \to \left( \left( \nabla^2 - \frac{1}{c^2} \frac{\partial^2}{\partial t^2} \right)_g S_Q \right) \tag{100}$$

or

$$L_{quantum} = \frac{1}{\sqrt{(\nabla_\mu S_Q)_g^2 - \left( \nabla^2 - \frac{1}{c^2} \frac{\partial^2}{\partial t^2} \right)_g S_Q}} \to \infty \tag{101}$$

which provide a sort of a correspondence principle in the 3D quantum vacuum described by the generalized Klein-Gordon equation. Different values of the quantum length (99) correspond to different types, or degrees, of correlations at the fundamental arena represented by the 3D quantum vacuum described by the wave functions (80). This turns out to be in agreement with the recent proposal that, in the quantum domain, "all entangled quantum states are non-local" [63].

With the introduction of the quantum entropy (96) – defining the degree of order and chaos of the 3D quantum vacuum – which leads directly to the quantum length (99), one can thus throw new light on the interpretation of non-locality in the quantum domain. In fact, on the basis of the features of the quantum length (99), one can say that, as a consequence of the two quantum correctors $-\frac{c^2\hbar^2}{V^2\Delta\rho^2} (\nabla_\mu S_Q)_g^2$ and $\frac{c^2\hbar^2}{V^2\Delta\rho^2} \left( \left( \nabla^2 - \frac{1}{c^2} \frac{\partial^2}{\partial t^2} \right)_g S_Q \right)$ the 3D quantum vacuum acts as an immediate information medium in the behaviour of the changes of the quantum vacuum energy density determining the appearance of a spinless particle. The non-local action of the quantum potential associated with the wave function of the quantum vacuum for the generalized Klein-Gordon equation derives just from the quantum length (99) and thus from the quantum entropy (96). In other words, one can say that by introducing the quantum entropy given by equation (96), it is just the two quantum correctors above mentioned, depending on the quantity describing the degree of order and chaos of the vacuum supporting the density $\rho$ (of the elementary quasi-particles associated with the wave function of the quantum vacuum under consideration) the ultimate elements which, at a fundamental level, produces an immediate information medium in the behaviour of the changes of the quantum vacuum energy density determining the appearance of a spinless particle. Moreover, in this picture, the space we perceive seems to be characterized by local features because in our macroscopic domain the quantum vacuum entropy satisfies condition (100) (or, that is the same, relation (101)).

In synthesis, one can say that the quantum vacuum entropy (96) can be indeed interpreted as a sort of a non-local intermediary entity between the background and the appearance of spinless quantum particles. The quantum vacuum entropy can be interpreted as the fundamental element which gives origin to the non-local action of the wave function of the quantum vacuum (and the quantum length (99) suggests that there are indeed various degrees of non-locality depending of its value).

In the context of our 3D quantum vacuum model, now let us make some considerations on how one can develop a mathematical treatment of particles of spin ½ in terms of the quantum vacuum energy density. To achieve this, one can start by writing the wave function of the quantum vacuum as

$$\psi_S(x) = \psi_S^{(P)}(x) + \psi_S^{(A)}(x) \tag{102}$$

where $\psi_S^{(P)}(x)$ represents the wave function of the excited states of the quantum vacuum associated with the appearance of particles of spin ½ and $\psi_S^{(A)}(x)$ represents the wave function of the excited states of the quantum vacuum associated with the appearance of the corresponding antiparticles. These two set of wave functions of the quantum vacuum can be expanded as

$$\psi_S^{(P)}(x) = \sum_k b_k u_k(x) \tag{103}$$

$$\psi_S^{(A)}(x) = \sum_k d_k^* v_k(x) \tag{104}$$

respectively [64-66]. Here $u_k$ are positive-frequency 4-spinors of the quantum vacuum while $v_k$ are negative frequency 4-spinors of the quantum vacuum; they together form a complete set of orthonormal solutions to the non-linear generalized Dirac equation for the quantum vacuum energy density

$$\left(i\gamma^\mu \partial_\mu - \frac{V \cdot \Delta\rho}{c\hbar}\right)\psi_S = 0 \tag{105}$$

where $x = \left(x^0, x^1, x^2, x^3\right) = (t, x)$, $\gamma_\mu$ are the well-known relativistic matrices $\gamma^0 = 1 \otimes \begin{pmatrix} 1 & 0 \\ 0 & -1 \end{pmatrix}$, $\gamma^i = \sigma_i \otimes \begin{pmatrix} 0 & 1 \\ -1 & 0 \end{pmatrix}$ and $\sigma$ are the Pauli matrices.

In equations (103) and (104), the label $k$ is an abbreviation for the set $(k, s)$ where $k$ is the 3-momentum $k = (p_1, p_2, p_3)$ and $s = \frac{1}{2}\hbar$, is the label of spin. As regards the expressions of $u_k$ and $v_k$, equation (105) leads to the following results:

$$u = W^z(k) \exp\left[-\frac{i}{\hbar} p_\mu x^\mu\right] \tag{106}$$

with z=1,2 and

$$v = W^z(k) \exp\left[\frac{i}{\hbar} p_\mu x^\mu\right] \tag{107}$$

with z=3,4, where

$$W^1 = \sqrt{\frac{E + V \cdot \Delta\rho}{2V \cdot \Delta\rho}} \begin{bmatrix} 1 \\ 0 \\ \frac{p_3 c}{E + V \cdot \Delta\rho} \\ \frac{(p_1 + ip_2)c}{E + V \cdot \Delta\rho} \end{bmatrix} \tag{108}$$

$$W^2 = \sqrt{\frac{E + V \cdot \Delta\rho}{2V \cdot \Delta\rho}} \begin{bmatrix} 0 \\ 1 \\ \frac{(p_1 - ip_2)c}{E + V \cdot \Delta\rho} \\ \frac{-p_3 c}{E + V \cdot \Delta\rho} \end{bmatrix} \tag{109}$$

$$W^3 = \sqrt{\frac{E + V \cdot \Delta\rho}{2V \cdot \Delta\rho}} \begin{bmatrix} \frac{p_3 c}{E + \frac{4}{3}\pi R^3 \Delta\rho} \\ \frac{(p_1 + ip_2)c}{E + V \cdot \Delta\rho} \\ 1 \\ 0 \end{bmatrix} \tag{110}$$

$$W^4 = \sqrt{\frac{E + V \cdot \Delta\rho}{2V \cdot \Delta\rho}} \begin{bmatrix} \frac{(p_1 - ip_2)c}{E + V \cdot \Delta\rho} \\ \frac{-p_3 c}{E + V \cdot \Delta\rho} \\ 0 \\ 1 \end{bmatrix} \tag{111}$$

where $E$ is the energy of the particle determined by the excited state of the quantum vacuum characterized by change of energy density (23).

On the basis of equation (105) it becomes permissible the following re-reading of the behaviour of subatomic particles of spin $1/2$: subatomic particles of spin $1/2$ satisfying the standard Dirac equation derive from excited states of the 3D quantum vacuum described by wave functions of the form (106) and (107) which satisfy the more fundamental generalized

Dirac equation for the quantum vacuum energy density (105). In other words, inside the approach of the quantum vacuum energy density here proposed, relativistic quantum mechanics based on the standard Dirac equation can be considered as an emergence of the properties of more fundamental excited states of the 3D quantum vacuum associated with opportune changes of the quantum vacuum energy density which satisfy a generalized Dirac equation (105).

Another important problem to treat inside the approach based on the generalized Dirac equation regards the link with the curvature of space-time. The question concerning how to generalize Klein-Gordon and Dirac equations on the case of curved space-time was originally faced by Fock [67] and Weyl [68]. More recently, interesting perspectives have been suggested by A. D. Alhaidari and A. Jellal. In their recent article *Dirac and Klein-Gordon equations in curved space* Alhaidari and Jellal provided a new systematic formulation of the standard Dirac and Klein-Gordon equations of quantum theory in a curved space by introducing a matrix operator algebra involving the Dirac gamma matrices with a universal length scale constant as a measure of the curvature of space [69]. In this way, they found that spin connections or vierbeins are no longer required in order to derive the Dirac equation and that the Klein-Gordon equation emerges in its canonical form without first order derivatives. Alhaidari's and Jellal's research introduce the perspective to unify in one single scheme the generalized Klein-Gordon and Dirac equations for the quantum vacuum energy density.

In the final part of this chapter, by following the philosophy that is at the basis of Alhaidari's and Jellal's approach, we want to develop a sort of unification of the generalized Klein-Gordon and Dirac equations for the quantum vacuum energy density in a scheme based on a matrix operator algebra of the quantum vacuum energy density describing the curvature of space.

The unitary picture of the generalized Klein-Gordon and Dirac equations for the quantum vacuum energy density is given by the following version of the generalized Dirac equation for the wave function of the 3D quantum vacuum:

$$\left(i\gamma^{\mu}\partial_{\mu}+\lambda\beta\right)\psi_{S}=\frac{V\cdot\Delta\rho}{\hbar c}\psi_{S} \tag{112}$$

where $\beta$ is a space dependent matrix, $\lambda$ is a universal real constant of inverse length dimension that gives a measure of the curvature of space (e.g., the inverse of the "effective radius of curvature" of space). Iteration of equation (112) (namely, squaring this equation) gives

$$\left[-\gamma^{\mu}\gamma^{\nu}\partial_{\mu}\partial_{\nu}+\left(-\gamma^{\mu}\partial_{\mu}\gamma^{\nu}+i\lambda\left\{\beta,\gamma^{\nu}\right\}\right)\partial_{\nu}+i\lambda\gamma^{\mu}\partial_{\mu}\beta+\lambda^{2}\beta^{2}\right]\psi_{S} = \left(\frac{V\cdot\Delta\rho}{\hbar c}\right)^{2}\psi_{S} \tag{113}$$

which yields directly the generalized Klein-Gordon equation under appropriate conditions for the matrix $\beta$. In fact, by defining $\hat{D}=\gamma^{\mu}\partial_{\mu}$ and by requiring that

$$-\frac{i}{\lambda}\hat{D}\gamma^{\mu}=\{\beta,\gamma^{\mu}\} \tag{114}$$

$$-\frac{i}{\lambda}\hat{D}\beta=\{\beta,\beta\}=\beta^{2} \tag{115}$$

equation (113) becomes

$$\left(g^{\mu\nu}\partial_{\mu}\partial_{\nu}+\lambda^{2}\beta^{2}-\frac{V^{2}\left[\Delta\rho\right]^{2}}{\hbar^{2}c^{6}}\right)\psi_{S}=0 \tag{116}$$

which is the generalized Klein-Gordon equation for the quantum vacuum energy density in a curved space in its simple canonical form (namely with no first order derivatives). It is thus required that $\beta^{2}$ be diagonal so that no component coupling (spin coupling) is present in the generalized Klein-Gordon equation. Moreover, it is possible to extend the $n+2$ dimensional algebra defined by equations (114) and (115), and which is defined for a given positive integer $n$ by the matrix $\beta$ and the metric $g^{\mu\nu}$, to any square matrix that belongs to the space of matrices of our consideration. That is to say, any matrix $\alpha$ which is an element of this algebra must satisfy the following condition

$$-\frac{i}{\lambda}\hat{D}\alpha=\{\beta,\alpha\} \tag{117}$$

Therefore, equation (112) with the $n+2$ matrices $\{\beta,\gamma^{\mu}\}_{\mu=0}^{n}$ satisfying the matrix algebra conditions (114) and (115) can be considered here as the generalized Dirac equation for the quantum vacuum energy density in a curved space whose metric is $g^{\mu\nu}$ and curvature parameter is $\lambda$. The corresponding generalized Klein-Gordon equation for the quantum vacuum energy density in a curved space (which determines the appearance of a spin-0 particle) is equation (116). One of the advantages of the algebra defined by equations (114) and (115) is the absence of first order derivatives in the resulting generalized Klein-Gordon equation. Consequently, by disposing of the spin connections and frame fields in favour of the matrix operator algebra (117) has determined a generalized Dirac equation (112) in a curved space that leads naturally to the simple canonical form of the generalized Klein-Gordon equation (116) in a curved space.

About the problem of the unification of the generalized Klein-Gordon and Dirac equations, the link between the generalized Dirac equation for the quantum vacuum energy density and the curvature of space-time and the consequent problem of unifying gravity and quantum theory inside a single unitary mathematical formulation (which contains both the generalized Klein-Gordon and Dirac equations for the quantum vacuum energy density), further research will give you more information.

## 6 Conclusions

In this article a model of a background of physical processes in terms of a 3D quantum vacuum based on Planck energy density as universal property defining the ground state of the same flat-space arena has been proposed. By considering a Planckian metric and a variable energy density of quantum vacuum (which can be considered important novelties of this approach with respect to other theories about these topics), this model introduces interesting and relevant monistic perspectives in the interpretation of gravity and of quantum mechanics. On one hand, as the authors already obtained in the recent paper [55], within this model the changes of the quantum vacuum energy density allow us to reproduce the space-time curvature characteristic of general relativity and produce a shadowing of the gravitational space which influences the motion of the objects in a way that is coherent with the results of general relativity (in particular, about the weak field limit conditions of Solar system and the bending of light rays from a distant star passing near a massive body). On the other hand, and this is a striking novel result obtained in this paper, by making use of a first quantization scheme this model provides an interesting interpretation of the behaviour of spinless quantum particles, in terms of a generalized Klein-Gordon equation for the quantum vacuum energy density, which can show that the gravitational and the quantum effects of matter are highly coupled and have the same origin, namely the same 3D quantum vacuum characterized by opportune changes of its energy density. In this way, inside a picture where a actual variable energy density of quantum vacuum is the origin of mass and curvature of space, this model allows us to interpret in a single unifying picture important foundational ideas and results of two significant toy models of gravity recently developed, namely Consoli's model about physical vacuum as a superfluid medium characterized by density fluctuations of its elementary quanta and F. Shojai's and A. Shojai's toy model about the behaviour of spinless particles in a curved space-time.

Moreover, on the basis of the model of the 3D quantum vacuum energy density here suggested, the interesting perspective is opened to read in a unifying way gravitation and quantum effects as two different aspects of the same source, namely the wave functions of opportune excited states of the quantum vacuum. The excited states of the 3D quantum vacuum (corresponding to opportune changes of the quantum vacuum energy density) can be considered as the real bridges between gravitational effects and quantum effects of matter and are ruled by a quantum potential which determines a fundamental non-local character of the processes. As regards the treatment of the wave functions of the 3D quantum vacuum determining the appearance of spinless particles, the interesting perspective is opened that the non-locality emerges from a fundamental quantum entropy describing the degree of order and chaos of the background supporting the density of the ensemble of elementary structures associated with the wave function of the quantum vacuum. Finally, a mathematical treatment of particles of spin $1/2$ based on a generalized Dirac equation for the quantum vacuum energy density and an unification of the generalized Klein-Gordon equation and Dirac equation (in a scheme based on a matrix operator algebra of the quantum vacuum energy density describing the curvature of the background) have been suggested.

Nonetheless, the approach proposed in this article places some important questions, in particular the matter regarding the explanation of quantum non-locality versus classical non-locality and the consequent explanation of how local structures emerge from elementary vibrating entities described by wave functions of the quantum vacuum characterized by a non-local information. In fact, it must be emphasized that, as regards quantum non-locality, when entangled particles are present the Bohm dynamics is usually formulated in the configuration space, not in the 3D physical space, whilst in the approach here suggested the quantum vacuum energy density, as ultimate origin of quantum non-locality, is assumed to be a density in a 3D space. Here, the fundamental perspective is indeed introduced that quantum non-locality regarding the ordinary quantum level of subatomic particles emerges from a more fundamental non-locality of the vacuum associated to the spatial inhomogeneity of the vacuum, which, in the light of equations (95)-(98), and in particular the features of the quantum length (98), makes the 3D quantum vacuum an immediate information medium in the behaviour of the changes of the quantum vacuum energy density determining the appearance of a spinless particle, as a consequence of the two quantum correctors $-\frac{c^2\hbar^2}{V^2\Delta\rho^2}\left(\nabla_\mu S_Q\right)_g^{\ 2}$ and $\frac{c^2\hbar^2}{V^2\Delta\rho^2}\left(\left(\nabla^2-\frac{1}{c^2}\frac{\partial^2}{\partial t^2}\right)_g S_Q\right)$. At a fundamental level, one can say that in quantum vacuum time has only mathematical existence, namely, time is the numerical order of motion. In quantum vacuum it is always NOW, there is no past, present or future and thus the perspective is opened that indeed quantum vacuum itself is the medium of immediate information transfer both in classical and quantum non-locality (in the sense that both classical and quantum non-locality should ultimately emerge from the more fundamental non-locality characterizing the 3D quantum vacuum). In this regard, of course further investigations will be needed to clarify these connections.

Finally, another important problem regards the eventual possibility to discriminate experimentally the view proposed in this paper. In particular, a relevant novelty suggested by this approach lies in the fact that gravity is not a propagating force, but is immediate, the result of variable energy density of quantum vacuum. As a consequence, under this point of view, idea of graviton as physical entity transporting gravity should be superfluous. The question of checking if the immediate action of gravity leads (or does not lead) to new observable effects in the solar system is open.

However, despite the questions regarding the possible discrimination of the view of gravity proposed here and the problem of the explanation of quantum non-locality and classical non-locality, and of how locality and local structures of the macroscopic world emerge from a fundamental non-local 3D quantum vacuum, as well as the unification of gravity with quantum theory inside this approach (which requires further research investigation), from an epistemological point of view, it is relevant that in the model here proposed it is possible to provide an unification of Heraclitean and Parmenedian aspects of contemporary physics. In fact, being and becoming can be indeed considered as two different aspects of the same coin, namely one obtains a peculiar unifying complementarity in which the becoming of the events, of motion and evolution derives from the primary physical reality represented by elementary fluctuations of the quantum vacuum energy density and, at the same time, neither geometry nor algebra may be considered as fundamental in physics: there is only a monistic principle corresponding to the elementary changes of quantum vacuum energy density [70].

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