Gravitation, cosmology and space-time torsion

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ABSTRACT. The Poincaré gauge theory of gravity offers opportunities to solve some principal problems of general relativity theory and modern cosmology. In the framework of this theory the gravitational interaction can have both a repulsive character, as well as the usual attractive character found in gravitating matter with positive values of energy density and pressure satisfying the energy dominance condition. Cosmological consequences of gravitational repulsion are considered in the case of homogeneous isotropic models in connection with the problem of cosmological singularity and dark energy problem of general relativity theory. Regular Big Bang inflationary scenario with an accelerating stage of cosmological expansion at asymptotics are discussed in terms of the principal role of played by space-time torsion.

1 Introduction

Einsteinian general relativity theory (GR) is the base of modern theory of gravitational interaction and relativistic cosmology. GR allows to describe different gravitating systems and cosmological models at widely changing scales of physical parameters. At the same time GR possesses certain principal problems, which, in particular, appear in cosmology. One of the most principal cosmological problems is the problem of cosmological singularity (PCS). Cosmological solutions of GR describing the evolution of the Universe have the beginning in the time, and in accordance with Einstein gravitation equations the singular state with divergent energy density and singular metrics appears at the beginning of cosmological expansion. It is because the gravitational interaction for
usual gravitating matter with positive values of energy density and pressure satisfying energy dominance condition in the frame of GR as well as Newton's theory of gravity has the character of attraction, but never repulsion. The PCS is particular case of general problem of gravitational singularities of GR [1]. Note that in the frame of GR the gravitational interaction can have the repulsion character in the case of gravitating systems with negative pressure (for example, scalar fields in inflationary cosmology). However, the PCS can not be solved by taking into account such systems. According to widely known opinion, the solution of PCS has to be connected with quantum gravitational effects beyond Planckian conditions, when the energy density surpasses the Planckian one. A number of particular regular cosmological solutions was obtained in the frame of candidates to quantum gravitation theory – string theory/M-theory and loop quantum gravity. Radical ideas connected with notions of strings, branes, extra-dimensions, space-time foam etc are used in these theories (some features of these solutions are discussed in [2,3]).

As it was shown in a number of papers (see [2-4] and Refs herein) the gauge approach in theory of gravitational interaction offers opportunities to solve the PCS in the frame of usual field-theoretical description of gravity in 4-dimensional physical space-time, where constructive Einsteinian definitions of space-time notions are valid locally. The structure of physical space-time in the framework of gauge approach to gravitation, generally speaking, is more complicated in comparison with GR. So, in the frame of the Poincaré gauge theory of gravity (PGTG), which is the most important gauge theory of gravitation, the physical space-time possesses the structure of Riemann-Cartan continuum. Gravitational field in PGTG is described by means of interacting metric and torsion fields. The presence of space-time torsion can change the character of gravitational interaction by certain physical conditions imposed on the case of usual gravitating matter. This fact enables the solution of some principal problems of GR including the PCS.

The present paper is organized by the following way. In Section 2 the question "why we need the Poincaré gauge theory of gravity" is discussed. Principal remarks concerning the solution of PCS in the frame of PGTG are given in Section 3, and in Section 4 recent results about a possible solution of the "dark energy problem" of GR in terms of PGTG are briefly discussed. In Conclusion some possible physical consequences of space-time torsion in connection with other problems of GR are discussed.
2 Local gauge invariance principle and Poincaré gauge theory of gravity

As it is known, the local gauge invariance principle is the basis of modern theory of fundamental physical interactions. The theory of electro-weak interaction, quantum chromodynamics, Grand Unified models of particle physics were built by using this principle. From physical point of view, the local gauge invariance principle establishes the correspondence between certain important conserving physical quantities, connected according to the Noether’s theorem with some symmetry groups, and fundamental physical fields. These fields are presumed to have as a source the corresponding physical quantities, and play the role of carriers of fundamental physical interactions. The application of this principle to gravitational interaction leads to generalization of Einstein theory of gravitation.

The local gauge invariance principle was applied by Utiyama in Ref.[5] in order to build a theory of gravitation by considering the Lorentz group as gauge group corresponding to gravitational interaction. Utiyama introduced the Lorentz gauge field, which has transformation properties of anholonomic Lorentz connection. By identifying this field with anholonomic connection of Riemannian space-time, Utiyama obtained Einstein gravitation equations of GR. The work by Utiyama [5] was criticized by many authors. First of all, if anholonomic Lorentz connection is considered as independent gauge field, it can be identified with a connection of Riemann-Cartan continuum with torsion, but not a Riemannian connection [6-8]. Moreover, if a source of gravitational field includes the energy-momentum tensor of gravitating matter, we can not consider the Lorentz group as gauge group corresponding to gravitational interaction. Note that metric theories of gravitation in 4-dimensional pseudo-Riemannian space–time including GR, in the frame of which the energy-momentum tensor is a source of gravitational field, can be introduced in the frame of gauge approach by the localization of 4-parametric translation group [9, 10]. By localizing 4-translations and introducing gauge field as symmetric tensor field of second order, the structure of initial flat space-time changes, and gauge field becomes

\[ \text{In the gauge approach, the gravitation interaction is connected with space-time transformations. Hence the gauge treatment to gravitation has essential differences in comparison with Yang-Mills fields which are connected with internal symmetries groups. As a result, there are different gauge treatments to gravitational interaction not detailed in this paper.} \]
connected with metric tensor of physical space-time. As the localized translation group leads us to general coordinate transformations, from this point of view the general covariance of GR plays the dynamical role. At the same time the local Lorentz group (group of tetrad Lorentz transformations) in GR and other metric theories of gravitation does not play any dynamical role from the point of view of gauge approach, because the corresponding Noether invariant in these theories is identically equal to zero \[11\]. The other treatment to localization of translation group was presented in \[12, 13\], where gravitation field was introduced as tetrad field in 4-dimensional space-time with absolute parallelism. This theory is not covariant with respect to localized tetrad Lorentz transformations, and in fact it is intermediate step to gravitation theory with independent gauge Lorentz field. If one means that the Lorentz group plays the dynamical role in the gauge field theory and the Lorentz gauge field exists in the nature, we obtain necessarily the gravitation theory in the Riemann-Cartan space-time as natural generalization of GR (see, for example, \[14-16\]). The corresponding theory is known as Poincaré gauge theory of gravitation.

Gravitational gauge field variables in PGTG are the tetrad \(h^i_\mu\) (translational gauge field) and the Lorentz connection \(A^{ik}_\mu\) (Lorentz gauge field); corresponding field strengths are the torsion tensor \(S^i_\mu\nu\) and the curvature tensor \(F^{ik}_\mu\nu\) defined as

\[
S^i_\mu\nu = \partial_\nu h^i_\mu - h^i_\kappa A^{ik}_\mu \],
\[
F^{ik}_\mu\nu = 2\partial_\nu A^{ik}_\mu + 2A^{ij\mu}_\nu A^{k}_\xi \bigg|_{\xi = \nu} ,
\]

where holonomic and anholonomic space-time coordinates are denoted by means of greek and latin indices respectively. In the case of gravitating matter minimally coupled with gravitation, the sources of gravitational field in PGTG are the energy-momentum and spin tensors.

The gravitational Lagrangian of PGTG is invariant built by means of gravitational field strengths. The simplest PGTG is the Einstein-Cartan theory based on gravitational Lagrangian in the form of scalar curvature of Riemann-Cartan space-time \[7,8,17\]. In certain sense the Einstein-Cartan theory of gravitation is a degenerate theory \[18\]. Like gauge Yang-Mills fields, the gravitational Lagrangian of PGTG has to include invariants quadratic in gravitational field strengths - curvature and torsion tensors. Inclusion of a linear in curvature term (scalar curvature) to the gravitational Lagrangian is necessary to satisfy the correspondence principle with GR.
We will consider the PGTG with gravitational Lagrangian $L_G$ given in general form containing different invariants quadratic in the curvature and torsion tensors

$$L_G = f_0 F + F^\alpha \beta \mu \nu (f_1 F_{\alpha \beta \mu \nu} + f_2 F_{\alpha \mu \beta \nu} + f_3 F_{\mu \nu \alpha \beta}) + F^\mu \nu (f_4 F_{\mu \nu} + f_5 F_{\nu \mu})$$

$$+ f_6 F^2 + S^\alpha \mu \nu (a_1 S_{\alpha \mu \nu} + a_2 S_{\nu \mu \alpha}) + a_3 S^\alpha_{\mu \nu} S_{\beta \rho \delta},$$

where $F_{\mu \nu} = F_{\alpha \mu \alpha \nu}$, $F = F_{\mu \mu}$, $f_i$ ($i = 1, 2, \ldots, 6$), $a_k$ ($k = 1, 2, 3$) are indefinite parameters, $f_0 = (16\pi G)^{-1}$, and $G$ is Newton’s gravitational constant (the light velocity in the vacuum $c = 1$). Although the gravitational Lagrangian (1) includes a number of indefinite parameters, gravitational equations of PGTG for homogeneous isotropic models (HIM) considering below depend weakly on the choice of quadratic part of gravitational Lagrangian by virtue of their high spatial symmetry.

3 Problem of cosmological singularity and PGTG

According to observational data concerning anisotropy of cosmic microwave background, our Universe was sufficiently homogeneous and isotropic during the initial stages of cosmological expansion. In connection with this fact, the investigation of HIM is of greatest interest for relativistic cosmology. In the frame of PGTG homogeneous isotropic models are described in general case by means of three functions of time: the scale factor of Robertson-Walker metrics $R(t)$ and two torsion functions $S_1(t)$ and $S_2(t)$ determining the following components of torsion tensor (with holonomic indices) [19]:

$$S_{10} = S_{20} = S_{30}, S_{11} = S_{12} = S_{23} = S_{31} = S_{1} (t), S_{123} = S_{231} = S_{312} = S_2 (t) \frac{R^3}{(1 - kr^2)^2} \sin \theta,$$

where spatial spherical coordinates are used and $k = +1, 0, -1$ for closed, flat and open models respectively. The functions $S_1$ and $S_2$ have different properties with respect to transformations of spatial inversions, namely, unlike $S_1$ the function $S_2$ has pseudoscalar character.

At first we will consider HIM with vanishing pseudoscalar torsion function (see [19, 2-4] and references herein) filled by spinless gravitating matter with energy density $\rho$ and pressure $p$. In this case gravitational equations of PGTG lead to the following generalized cosmological Friedmann equations (GCFE):

$$\frac{k}{R^2} + \left\{ \frac{d}{dt} \ln \left[ R \sqrt{1 + \alpha (\rho - 3p)} \right] \right\}^2 = \frac{8\pi G}{3} \frac{\rho + \frac{\alpha}{4} (\rho - 3p)^2}{1 + \alpha (\rho - 3p)},$$

(2)
\[ R^{-1} \frac{d}{dt} \left[ \frac{dR}{dt} + R \frac{d}{dt} \left( \ln \sqrt{|1 + \alpha (\rho - 3p)|} \right) \right] = - \frac{4\pi G}{3} \frac{\rho + 3p - \frac{\alpha}{2} (\rho - 3p)^2}{1 + \alpha (\rho - 3p)}, \]  

where indefinite parameter \( \alpha = \frac{f}{3f_0^2} > 0 \) \((f = f_1 + \frac{f_2}{2} + f_3 + f_4 + f_5 + 3f_6)\) has the inverse dimension of energy density \(^2\).

According to gravitational equations the torsion function \( S_1 \) is

\[ S_1 = - \frac{1}{4} \frac{d}{dt} \ln |1 + \alpha (\rho - 3p)| \]  

and conservation law for gravitating matter has usual form

\[ \dot{\rho} + 3H (\rho + p) = 0, \]  

where \( H = \dot{R}/R \) is the Hubble parameter and a dot denotes the differentiation with respect to time.

If the parameter \( \alpha \) tends to zero, the torsion function (4) vanishes and GCFE (2)-(3) coincide with the Friedmann cosmological equations of GR. The difference of (2)–(3) from the Friedmann cosmological equations of GR is connected with the terms containing the parameter \( \alpha \). The value of \( \alpha^{-1} \) determines the scale of extremely high energy densities. The solutions of GCFE coincide practically with corresponding solutions of GR, if the energy density is small \(|\alpha (\rho - 3p)| \ll 1 \) \((p \neq \frac{1}{3} \rho)\). The difference between GR and PGTG can be significant at extremely high energy densities \(|\alpha (\rho - 3p)| \gtrsim 1\), where the dynamics of HIM depends essentially on the space-time torsion \(^3\).

The structure of GCFE (2)–(3) ensures regular behavior of cosmological solutions. In order to demonstrate this fact in the case of inflationary cosmological models, we will consider below HIM filled with scalar field \( \phi \) minimally coupled with gravitation and gravitating matter with equation of state in the form \( p_m = p_m(\rho_m) \) (values of gravitating matter are

\(^2\)The second indefinite parameter \( a = 2a_1 + a_2 + 3a_3 \) connected with quadratic in the torsion part of Lagrangian (1) in gravitational equations for HIM has to be equal to zero, if one supposes that cosmological equations do not contain high derivatives with respect to the scale factor \( R(t) \).

\(^3\)Ultrarelativistic matter \((p = \frac{1}{3} \rho)\) and gravitating vacuum \((p = -\rho)\) with constant energy density are two exceptional systems, because GCFE (2)–(3) are identical to the Friedmann cosmological equations of GR in these cases independently on values of the energy density.
denoted by means of index “m”). Then the energy density $\rho$ and the pressure $p$ take the form

\[ \rho = \frac{1}{2} \phi^2 + V + \rho_m \quad (\rho > 0), \quad p = \frac{1}{2} \phi^2 - V + p_m, \tag{6} \]

where $V = V(\phi)$ is a scalar field potential. As the energy density $\rho$ is positive and $\alpha > 0$, from equation (2) in the case $k = +1, 0$ it follows that:

\[ Z = 1 + \alpha (\rho - 3p) = 1 + \alpha \left( 4V - \dot{\phi}^2 + \rho_m - 3p_m \right) \geq 0. \tag{7} \]

The condition (7) is valid not only for closed and flat models, but also for cosmological models of open type ($k = -1$) [2]. The domain of admissible values of scalar field $\phi$, time derivative $\dot{\phi}$ and energy density $\rho_m$ determined by (7) is limited in the space $P$ of these variables with boundary $L$ defined as

\[ Z = 0 \quad \text{or} \quad \dot{\phi} = \pm \left( 4V + \alpha^{-1} + \rho_m - 3p_m \right)^{\frac{1}{2}}. \tag{8} \]

Unlike GR at compression stage the time derivative $\dot{\phi}$ does not diverge, and by reaching the bound $L$ the transition to the second part of cosmological solution containing the expansion stage takes place. From cosmological equation (2) by using the conservation law (5) it follows that in the space $P$ there are extremum surfaces upon which the Hubble parameter vanishes [2]. Extremum surfaces play the role of “bounce surfaces”, because the time derivative of the Hubble parameter is positive on the greatest part of these surfaces in the case of scalar field potentials occurring in chaotic inflation [2,4]. All cosmological solutions have bouncing character and are regular with respect to metrics, Hubble parameter, its time derivative and energy density of gravitating matter. If gravitating matter satisfies standard conditions (energy density is positive, energy dominance condition is valid), any cosmological solution is not limited in time, and before the expansion stage the cosmological solution contains the compression stage and regular transition from compression to expansion. If the value of scalar field at the beginning of cosmological expansion is sufficiently large ($\phi \geq 1M_p$, where $M_p$ is the Planckian mass), the cosmological solution (similarly to corresponding solutions in GR) contains a quasi-de-Sitter inflationary stage and post-inflationary stage with oscillating scalar field. As the de-Sitter solution is an exact solution of GCFE [20], characteristics of the inflationary stage
in our theory (in particular, the duration of this stage by given initial conditions for scalar field at the beginning of expansion) are close to that of GR. As numerical analysis of the inflationary solutions of GCFE shows [4], the duration of transition stage from compression to expansion is several orders of magnitude less than the duration of inflationary stage. By taking into account that the duration of inflationary stage is extremely small [21], we can conclude that the regular cosmological solutions, discussed above, correspond to the regular Big Bang scenario or a Big Bounce. Note that if the scale of extremely high energy densities defined by $\alpha^{-1}$ is essentially less than the Planckian one, the behavior of the cosmological solution at the end of inflationary stage differs from that of GR (in particular, the Hubble parameter oscillates by changing its sign) [2,4]. After transformation of oscillating scalar fields into ultrarelativistic particles and the transition to radiation dominated stage, the further evolution of HIM (nucleosynthesis, transition to matter dominated stage) practically coincides with that of GR.

The regular character of all cosmological HIM described by GCFE is connected with a gravitational repulsion effect, where the principal role is played by space-time torsion [3]. Such a repulsion occurs in the theory of usual gravitating matter, with positive energy density, only at extreme conditions (extremely high energy densities and pressures).

4 Dark energy problem of GR and PGTG

Unlike the PCS, which is an old cosmological problem of GR, the dark energy problem (DEP) of GR is new problem which appeared together with the discovery of the acceleration of cosmological expansion at the present epoch. By using Friedmann cosmological equations of GR in order to explain accelerating cosmological expansion, the notion of dark energy (or quintessence) was introduced in cosmology. According to current estimates, approximately 70% of energy in our Universe is related with some hypothetical form of gravitating matter with negative pressure — dark energy — of unknown nature. Previously a number of investigations devoted to DEP were carried out (see review [22]). According to widely known opinion, the dark energy is associated with a cosmological term. If the cosmological term is related to the vacuum energy density, it is necessary to explain why it has the value close to the critical energy density at the present epoch (see for example [23]). Note that by including a cosmological term of comparable value to GCFE, we can build regular cosmology with an observable accelerating expansion
stage in the frame of PGTG. However, like GR, the DEP is not solved by this way.

As it was shown in Refs. [24,25], the PGTG offers opportunities to solve the DEP without using the notion of dark energy. It is because the space-time torsion in PGTG can change the character of gravitational interaction and lead to gravitational repulsion effect not only at extreme conditions, but also at very small energy densities. With this purpose the HIM with two torsion functions were built and investigated in the frame of PGTG. Cosmological equations for such HIM include the pseudoscalar torsion function \( S_2 \) with its first time derivative, and contain besides \( \alpha \) also two other indefinite parameters: \( b = a_2 - a_1 \) with dimension of parameter \( f_0 \) and dimensionless parameter, \( \varepsilon \), which is function of coefficients \( f_i \) of gravitational Lagrangian. The pseudoscalar torsion function \( S_2 \) satisfies a differential equation of second order. According to the gravitational equations, the function \( S_1 \) can be expressed as function of the Hubble parameter; the torsion function \( S_2 \) with its first time derivative and parameters characterizing gravitating matter. If one supposes that \( S_2 = 0 \), then the equation for \( S_2 \)-function vanishes, and the cosmological equations and the expression for \( S_1 \)-function take previous form given in Section 3. However, there is other solution with not vanishing function \( S_2 \). As was shown in Refs. [24,25], by certain restrictions on the indefinite parameters the cosmological equations lead to an asymptotic accelerating expansion stage, when the physical parameters characterizing cosmological models are sufficiently small. It is because the pseudoscalar torsion function has a non-zero constant value, asymptotically. For the case \( |\varepsilon| \ll 1 \):

\[
S_2^2 = \frac{f_0(f_0 - b)}{4fb} + \frac{\rho - 3p}{12b}.
\]

(9)

As a result the asymptotic cosmological equations take the form of the cosmological Friedmann equations with an effective cosmological constant induced by pseudoscalar torsion function:

\[
\frac{k}{R^2} + H^2 = \frac{1}{6b} \left[ \rho + 3 \left( \frac{f_0 - b}{f} \right)^2 \right],
\]

(10)

\[
\dot{H} + H^2 = -\frac{1}{12b} \left[ \rho + 3p - \frac{3(f_0 - b)^2}{2f} \right].
\]

(11)
By using the equation of state for dust matter at asymptotics, we find that the cosmological equations (10-11) lead to an observable accelerating cosmological expansion, if the indefinite parameters $b$ and $\alpha$ are connected by the following way:

$$b = [1 - (2.8\rho_{cr}\alpha)^{1/2}]f_0.$$ 

Note that the critical energy density is $\rho_{cr} = 6f_0H_0^2$ ($H_0$ is the value of the Hubble parameter at present epoch). If we suppose that the scale of extremely high energy densities defined by $\alpha^{-1}$ is larger than the energy density for quark-gluon matter, but less than the Planckian one, we obtain the corresponding estimation for $b$, which is very close to $f_0$.

The investigation of inflationary HIM with a pseudoscalar torsion function at extreme conditions at the beginning of cosmological expansion shows that the PGTG allows one to construct a totally regular inflationary Big Bang scenario [25]. Like HIM discussed in Section 3, there are extremum surfaces in space of independent variables $\phi, \dot{\phi}, S_2, \dot{S}_2, \rho_m$, upon which the Hubble parameter vanishes $H = 0$. Extremum surfaces depend on the indefinite parameters $\alpha, \varepsilon$ and in the case of open and closed models also on the scale factor $R$ (as was noted above, the value of $b$ depends on $\alpha$ and is close to $f_0$). Unlike HIM with a vanishing pseudoscalar torsion function, the bounce ($\dot{H}_0 > 0$) takes place only in limited domain of extremum surfaces with sufficiently small values of the function $S_2$. Properties of regular inflationary solutions with a pseudoscalar torsion function differ from that without pseudoscalar torsion function and depend essentially on indefinite parameters $\alpha$ and $\varepsilon$.

The regular Big Bang scenario was built in the frame of PGTG by classical description of gravitational field. If the energy density and values of torsion functions at the transition stage from compression to expansion are less than the Planckian ones, quantum gravitational era is absent during the evolution of the Universe. If the Planckian conditions were realized at the beginning of cosmological expansion, quantum gravitational corrections have to be taken into account; however, classical cosmological equations of PGTG ensure the regular character of the Universe evolution.

## 5 Conclusion

As follows from our consideration, the PGTG leads to certain principal differences in comparison with GR concerning the character of the gravitational interaction for usual gravitating matter that offers opportunities to solve some principal problems of GR. Although the direct interaction of the torsion with minimally coupled spinless matter is absent, corre-
sponding physical consequences of PGTG are connected essentially with space-time torsion by virtue of the interaction between metric and torsion fields. According to PGTG, the domain of applicability of GR is limited. Namely, in the case of cosmological HIM description, the domain of admissible energy densities has an upper limit determined by $\alpha^{-1}$ and lower limit equal to $\frac{\alpha(\alpha^2 - 1)}{\alpha}$. The following question appears: by what way the physical consequences of PGTG can be verified? As was noted above, the behavior of regular inflationary cosmological models at the end of inflationary stage, generally speaking, differs from that of GR and depends on indefinite parameter $\alpha$, and in the case of HIM with pseudoscalar torsion function also on parameter, $\varepsilon$. This can be a cause of possible differences of perturbations of scalar fields at the end of inflationary stage in comparison with GR, that has direct physical interest in connection with observable anomalies in anisotropy of cosmic microwave background [26]. This means that the development of a scalar fields perturbations theory in inflationary HIM in the frame of PGTG is of direct physical interest and possibly can test the cosmological consequences. The theoretical results can be important also for other gravitating systems in astrophysics. In particular, the conclusion about existence of limiting (maximum) energy density for gravitating systems can be significant for so-called primordial black holes limiting their admissible minimum mass $\frac{4}{\alpha}$. Together with dark energy problem, the problem of the origin of the non-baryonic component of dark matter is principal problem of relativistic cosmology and astrophysics. From our consideration of DEP given in Section 4 it follows that Newton’s law of gravitational attraction has limits of its applicability and space-time torsion can be essential in Newtonian approximation. If the torsion can lead to physical consequences in the frame of HIM as dark energy in GR, possibly the space-time torsion in the case of inhomogeneous matter distribution could be important for the solution of dark matter problem.

From our analysis given above follows that the PGTG can have the principal meaning for theory of gravitational interaction. Note that supergravity theory built in connection with the problem of unification of fundamental physical interactions, corresponds, strongly speaking, to PGTG, but not to a metric theory of gravitation, because the gauge group of supergravity theory includes the Lorentz group. As it is known,

\footnote{Note that the vacuum Schwarzschild solution for metrics with vanishing torsion is an exact solution of PGTG independently on indefinite parameters of gravitational Lagrangian (1).}
the simplest supergravity theory corresponds to the simplest PGTG – Einstein-Cartan theory. If the PGTG is a correct gravitation theory, then quantum gravitation theory must have as a quasi-classical approximation the gravitation theory in the Riemann-Cartan, but not pseudo-Riemannian space-time.

References


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